

# A New Maximum Fault-tolerance Barrier-coverage Problem in Hybrid Sensor Network and Its Polynomial Time Exact Algorithm

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## Abstract

This paper introduces a new maximum fault-tolerance barrier-coverage problem in hybrid sensor network, which consists of a number of both static ground sensors and fully-controllable mobile sensors. The problem aims to relocate the mobile sensor nodes so that the fault-tolerance of the barrier-coverage of the hybrid sensor network is maximized. The main contribution of this paper is the polynomial time exact algorithm for this new problem.

*Keywords:* Sensor networks, hybrid sensor networks, barrier-coverage, fault-tolerance, graph theory, optimal solution.

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## 1. Introduction

A wireless sensor node refers a microelectronic device which consists of a central processing unit, a wireless (usually radio) transceiver, a power source such as a battery, and most importantly one or more sensing units to observe designated events around it. Wireless sensor network consists of a number of low-cost wireless sensor nodes and is convenient to deploy for a wide range of application scenarios. Initially, wireless sensor network has been investigated for military applications. Recently, this innovative technology is also being

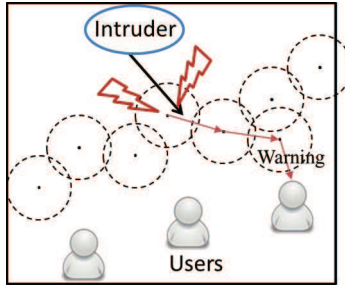


Figure 1: A wireless sensor network offers barrier-coverage over an area of interest if it can detect any designated trespasser who enters the area.

adopted to construct various civilian monitoring and surveillance applications with reasonable cost [2, 4, 6].

### 1.1. Full-coverage, Partial-coverage, and Barrier-coverage Models

In the literature, the term *coverage* of a wireless sensor network refers to the quality of surveillance service that the sensor network can offer. Based on the requirement, the coverage provided by wireless sensor network can be largely categorized into two categories. In detail, to provide *full-coverage* over an area of interest, a wireless sensor network should be able to monitor the entire area of interest concurrently [7, 8, 9, 10, 35, 5, 24]. On the other hand, *partial-coverage* can be provided by a wireless sensor network if the network does not meet the requirements to provide full-coverage but satisfies a certain requirement [11, 13, 14, 12]. *Barrier-coverage* is a partial-coverage model, which can be offered by a wireless sensor network over an area of interest if any intruder of interest who trespasses into the area from outside is guaranteed to be detected by the sensor network [15, 16, 17, 18] (see Fig 1). Due to its important applications such as enemy intrusion detection in the battlefield, barrier-coverage has received lots of attention recently.

### 1.2. Fault-tolerance Concept in Full-coverage and Barrier-coverage Models

In many deployment scenarios, a number of static wireless sensor nodes are randomly deployed over an area of interest and thus the sensor network has sufficient degree of coverage redundancy. Over years, many efforts are made to exploit the redundancy to improve the fault-tolerance as well as the lifetime of the coverage in the sensor network of cheap static ground sensors.

Within an area of interest covered by a wireless sensor network providing full-coverage with high redundancy, any target of interest within the area is likely to be observed by more than one sensor node at the same time. Specifically, by having the sensors in a way that every subregion in the area of interest is being observed by at least  $k$  sensors at the same time for some positive integer  $k$ , the full-coverage of the sensor network can provide up to any  $k - 1$  node failures. Naturally, by showing that a wireless sensor network has larger  $k$ , we can claim that the full-coverage of the sensor network has higher fault-tolerance against node failure. In the literature, one can show that the full-coverage of a wireless sensor network can endure up to  $k$  node failures by computing  $k$  node-disjoint subsets of the sensors such that each subset has enough sensors to monitor whole area of interest. In theory, the problem of computing maximum number of such subsets is known to be NP-hard and a number of approximation algorithms are introduced.

On the other hand, the goal of barrier-coverage is to monitor any intruder who trespasses into the area being surrounded by a sensor network. Therefore, to show that the barrier-coverage provided by a sensor network can endure up to  $k$  node failures, it is necessary to compute  $k$  node-disjoint subsets such that each subset can provide the desired barrier-coverage. Previously, Kumar et al. showed that the problem of computing the maximum number of node-disjoint sensor subsets can be modeled as the problem of computing the maximum number of node-disjoint paths in a graph induced by the sensor network and showed that the problem is polynomial time solvable [17].

### *1.3. Contributions and Outline of This Paper*

Thanks to the recent advances, various kind of mobile sensor nodes such as flying drones have been introduced. Motivated by such development, there have been a branch of researches which exploit mobile sensor nodes for surveillance or intrusion detection in the context of hybrid sensor networks, which consist of a few mobile sensor nodes and a number of traditional (cheap and ground) sensor nodes [19, 20, 21, 22, 1]. Since the cost of mobile sensor nodes are expected to be very high compared to the cost of the traditional sensor nodes, the main role of the mobile nodes tend to assist ground sensor nodes and reinforce their surveillance capability rather than constructing their own (mobile sensor nodes only) sensor network for surveillance.

**Main Contributions.** This paper introduces a new optimization problem

in hybrid wireless sensor network of both mobile and static sensors whose goal is to maximize the fault-tolerance of barrier coverage provided by the sensor network. We formally define this problem as the *maximum fault-tolerance barrier-coverage problem in hybrid sensor network (MFBP-HSN)*. Compare to the existing maximum fault-tolerance barrier coverage problem in (static) wireless sensor network, our problem requires to determine how to relocate the available mobile sensor nodes and to compute the maximum number of (static and mobile) node-disjoint paths at the same time. Apparently, this is a difficult problem as each mobile node can be positioned at any place on 2-dimensional Euclidean space and therefore the feasible solution space is infinite. To deal with the complexity, we first propose a new graph conversion technique to induce a new abstract graph from the original hybrid sensor network. Then, we use this graph as an input of our algorithm for MFBP-HSN. Most importantly, we show that this algorithm produces an optimal solution for MFBP-HSN and its running time is polynomial.

**Outline of The Paper.** The rest of this paper is organized as follows. Section 2 presents the related work and Section 4 provides some preliminaries. In Section 3, we introduce the formal definition of MFBP-HSN. Section 5 includes our main result, which is the polynomial time exact algorithm for the MFBP-HSN. Finally, we conclude this paper in Section 6.

## 2. Related Work

When it comes to the full-coverage of wireless sensor networks with static ground sensor nodes, the maximum fault-tolerance coverage problem is similar to the maximum lifetime coverage problem. This is because that one popular approach to maximize the lifetime of full-coverage of wireless sensor network of ground sensors is (a) computing the maximum number node-disjoint subsets such that each subset can full-cover the whole area of interest and (b) activating each subset one by one (activity one subset while the others are in a sleep mode until the nodes in the currently activated subset are exhausted) so that the total time period that the area of interest is maximized. Apparently, by activating multiple node-disjoint subsets at the same time, we can obtain higher level of fault-tolerance in full-coverage.

A sensor network is said to provide barrier-coverage over an area of interest if any trespasser which moves into the area is guaranteed to be detected by the sensor network. The barrier-coverage model is different from the tra-

ditional full-coverage model since it does not require to monitor the whole area of interest concurrently. As barrier-coverage model needs less number of nodes and is still effective to monitor any intruder, it is a very important concept for several main applications of wireless sensor networks such as intrusion detection. Originally, the concept of barrier-coverage has been introduced by Gage [30] in the context of robotic sensors. In [15], Kumar et al. introduced the concept of  $k$ -barrier-coverage. A sensor network provides  $k$ -barrier-coverage over an area of interest when an intruder moves into an area of interest, at least  $k$  sensors should be able to detect this. This means that  $k$ -barrier-coverage offers fault-tolerance against up to  $k - 1$  node failure. Since then, a considerable amount of attention has given to this model.

A  $k$ -barrier-coverage can be achieved by activating  $k$  node-disjoint subsets at the same time. In [15], Kumar et al. has modeled this as the problem of computing maximum number of node-disjoint paths in a graph which is induced from a given wireless sensor network of static nodes. This is a significant result because this problem of computing maximum number of node-disjoint paths in a graph is polynomial time solvable while the problem of maximum number of node-disjoint subsets such that each subset can fully cover the area of interest remains NP-hard. A distributed algorithm for this problem is introduced by Ban et al. [31]. Recently, Kim et al. [32] has shown that a particular type of attack can be launched by intruder if the consecutive operation schedule of barriers is not carefully designed.

As micro-electronic technologies have advances, a number of mobile sensor nodes such as drone flights are introduced. Since it is expected that the cost of mobile sensor nodes are significantly higher than the traditional ground sensor nodes, the main roles of the mobile sensor nodes are generally restricted to aiding the cheap ground sensor nodes and reinforce their coverage or extending their lifetime rather than constructing a wireless sensor network purely with mobile sensor nodes [19, 20, 21, 22, 3]. In [19], Wang et al. investigated the problem of computing the minimum number of mobile sensor nodes to help a given static ground sensor network to provide  $k$ -barrier-coverage. Ma et al. [20] has studied the problem of constructing  $k$ -barrier-coverage in hybrid sensor network of both mobile sensor nodes with limited mobility and static sensor nodes. Xu et al. [22] considered how to move available mobile sensor nodes so that the intruder detection ratio can be improved by using a single variable rst-order grey model, GM(1,1). In [21], Wang et al. investigated how to minimize the moving cost of mobile sensor nodes to fill the gap between sensor nodes and form a barrier. Note that

our problem is different from those existing work since our main goal is to form the strongest  $k$ -barrier-coverage in hybrid sensor networks by relocating available mobile nodes among pre-deployed static sensor nodes.

In [1], Kim et al. provided an important observation that the maximum lifetime barrier-coverage problem in hybrid sensor network completely different from the maximum fault-tolerant barrier-coverage problem in hybrid sensor network. Then, they focused on the first problem. In this paper, we focus on the second problem, which is defined as MFBP-HSN in this paper. We believe our problem is a dual of the problem in [19], which aims to determine the minimum number of mobile sensor nodes and their corresponding positions to achieve a required  $k$ -(hybrid)-barrier-coverage while the goal of our problem is to achieve the maximum fault-tolerance of  $k$ -(hybrid)-barrier-coverage provided by a given hybrid sensor network by determining the position of available mobile sensors. To the best of our knowledge, this problem has never been investigated so far.

### 3. Assumptions and Problem Definition

This paper assumes the ground sensor nodes as well as the mobile sensor nodes are homogenous, respectively. This means that all ground sensor nodes share the same hardware/software conditions, e.g. all sensors have the same initial battery level, the same sensing range, etc. It also implies that the sensing ranges of all mobile sensor nodes are same. As a result, the sensor network of our interest is a hybrid sensor network of two different kind of sensor nodes, static ground sensor nodes and mobile sensor nodes. Further, we assume that all sensor nodes are connected to the base station.

From now on, we will proceed our initial discussion based on the assumption that the sensing ranges of both mobile sensor nodes and static sensor nodes are same. Now, we provide the formal definition of our problem of interest.

**Definition 1** (MFBP-HSN). *Given a set  $V = \{v_1, \dots, v_n\}$  of static ground sensor nodes deployed over an area  $A$  of interest and  $l$  mobile sensor nodes, the maximum fault-tolerance barrier-coverage problem in hybrid sensor network (MFBP-HSN) is to determine the positions of the  $l$  available mobile sensor nodes so that the fault-tolerance of the barrier-coverage of the hybrid sensor network with  $l$  mobile sensors and  $n$  static sensors can be maximized.*

In this problem, we adopt the traditional assumption that an intruder must move across  $A$  to intrude and is not able to circumvent  $A$ . As we discussed earlier, MFBP-HSN is more challenging than the traditional maximum fault-tolerance barrier-coverage problem in pure static sensor network as we need to solve two co-related subproblems at the same time, that is, the problem of determining the positions of mobile sensors and the problem of computing  $k$ -node-disjoint hybrid sensor barriers.

#### 4. Preliminaries

Over decades, the maximum flow problem and its variations have attracted lots of attentions due to their significance in many applications. The inputs of a traditional maximum ( $s$ - $t$ ) flow problem instance include a given graph  $G = (V, E)$  with capacity on each edge in  $E$ , which represents the maximum amount of flow allowed to pass the edge, and two nodes  $s, t \in V$ . Then, the goal of the maximum flow problem is to compute the maximum amount of flow from  $s$  to  $t$  that the network can carry without violating the capacity limit on each edge. Let us introduce one important theorem.

**Theorem 1** ([17]). *Given  $G = (V, E)$ ,  $s, t \in V$ , and uniform capacity 1 for all edges, suppose we have a flow of amount  $k$  from  $s$  to  $t$  such that no node has incoming flow amount at most 1 and outgoing flow amount at most 1 (due to the constraint of maximum flow problem, they will be either 0 or 1 for both incoming and outgoing at the same time). Then, the resulting flow over  $G$  represents  $k$  node-disjoint paths from  $s$  to  $t$  in  $G$ .*

One important variation of the maximum  $k$ -flow problem is “the minimum cost  $\alpha$ -flow problem”<sup>1</sup>. The inputs of this problem include a graph  $G$  which has edge capacity as well as node weight (cost), two nodes  $s, t \in V$  and a required flow  $\alpha$ . The goal of this problem is to find the minimum (node) cost assignment (cost occurs at a node if any flow moves over the node) so that we can have  $\alpha$  flow to pass over the graph. It is known that this problem can be solved within polynomial time [33].

#### 5. A New Polynomial Time Optimal Algorithm for MFBP-HSN

In this section, we introduce our new algorithm for MFBP-HSN. We also prove that this algorithm produces the best possible solution within

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<sup>1</sup>In [33], this problem is simply referred as “the minimum cost flow problem”.

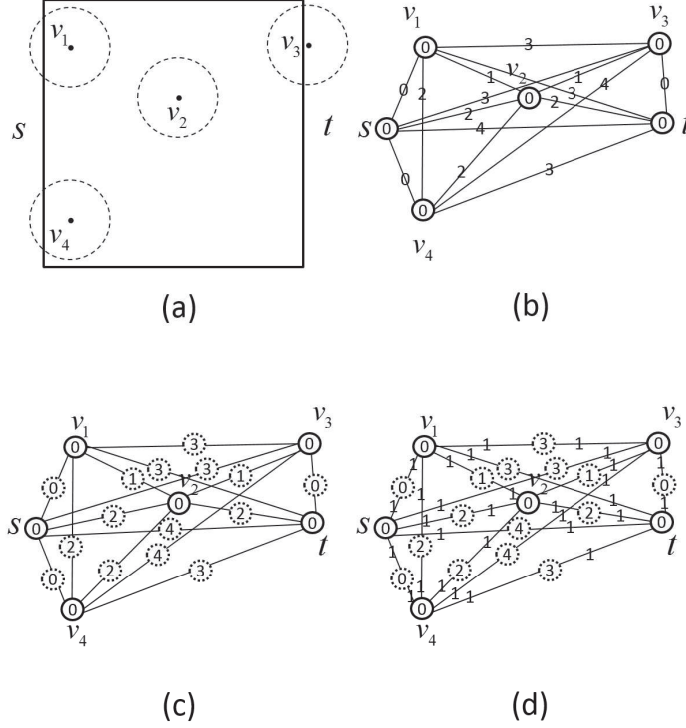


Figure 2: This figure illustrates how our algorithm works. Fig.(a) shows a given network instance. The graph in Fig.(b) shows the induced graph by Wang et al.’s strategy. Fig.(c) is the subdivision graph induced by our strategy. The node weight represents the number of mobile nodes we need to employ to construct a barrier cover including that node. In Fig.(d), we set the weight of each edge to be 1. Later, we convert this graph using Kumar et al.’s idea to make sure of node-disjointness of the resulting flow. Then, the only remaining task is to find a maximum flow whose cost is no greater than  $k$ , which is the number of available mobile nodes using a binary search strategy.

polynomial. The algorithm consists of the following 5 steps.

*Step 1. Construction of Initial Edge Weighted Graph  $\mathcal{G}$  [19]*

Given a set of  $n$  static ground sensors  $V = \{v_1, \dots, v_n\}$  and the area  $A$  over which the sensor nodes are deployed, we first construct a complete graph  $\mathcal{G}$  such that the set of nodes of  $\mathcal{G}$ ,  $V(\mathcal{G})$  is  $V \cup \{s, t\}$ , where  $s$  and  $t$  represent the area outside the left border and the area outside the right border of  $A$ , respectively (see Fig. 2(a)). As  $\mathcal{G}$  is a complete graph, there is an edge



between every pair of nodes in  $\mathcal{G}$ . For each pair of nodes  $v, u \in V(\mathcal{G})$ , we set the weight on each edge between  $v$  and  $u$  to be the minimum number of mobile sensor nodes to make a regional hybrid sensor barrier, which can be obtained by dividing the edge length with the diameter of the sensing range of a mobile node. Fig. 2(a) and Fig. 2(b) illustrate an example of this step.

*Step 2. Conversion to Node-weighted Graph  $\mathcal{G}'$*

Consider  $\mathcal{G}$  is the output of the Step 1. For each node in  $V(\mathcal{G})$ , set their node weight to be 0. Next, for each edge  $(u, v)$  in  $E(\mathcal{G})$ , we introduce a new node  $w$ , whose node weight is equivalent to the weight on the edge  $(u, v)$  and add it to  $V(\mathcal{G})$ . Then, remove  $(u, v)$  from  $E(\mathcal{G})$  and add two new edges  $(w, u)$  as well as  $(w, v)$  to  $E(\mathcal{G})$  whose edge weights are 0. Fig. 2(b) and Fig. 2(c) illustrate an example of this step. Let us define the final output node-weighted graph of this step as  $\mathcal{G}'$ . Observe that  $\mathcal{G}'$  is a node-weighted graph, but not an edge-weighted graph.

*Step 3. From MFBP-HSN to Maximum Node-disjoint  $s$ - $t$  Path Computation Problem*

Now, for each edge  $e$  in  $E(\mathcal{G}')$ , we set the capacity of  $e$  to be 1. Fig. 2(c) and Fig. 2(d) illustrate an example of this conversion. Suppose  $\mathcal{G}'$  becomes  $\hat{\mathcal{G}}$  after this conversion. Then,  $\hat{\mathcal{G}}$  has the following interesting properties.

**Property 1.** Consider a path from  $s$  to  $t$  (or  $s$ - $t$  path)  $P$  in  $\hat{\mathcal{G}}$ . Suppose  $X = \{s, v^{(1)}, v^{(2)}, \dots, t\}$  be the set of the nodes on the path  $P$ . By the construction rules of  $\hat{\mathcal{G}}$ , the sum of the weights on the nodes in  $X$  is equivalent to the minimum number of mobile nodes required to make one single hybrid barrier from  $s$  to  $t$  along with all the static nodes in  $X \setminus \{s, t\}$ . Note that this minimum number of needed mobile sensors can be easily calculated and is a function of the sensing ranges of the static and mobile sensor nodes.

**Property 2.** Consider a set of  $q$  node-disjoint  $s - t$  paths  $P_1, P_2, \dots, P_q$  in  $\hat{\mathcal{G}}$ . Suppose the sum of node weights on the  $q$  node-disjoint  $s - t$  paths is no greater than  $l$ . Then, by Property 1 and by the construction rules of  $\hat{\mathcal{G}}$ , the original hybrid sensor network is capable of providing at least  $q$ -barrier-coverage.

*Step 4. Computing Maximum Possible Node-disjoint Hybrid Barriers using Binary Search*

We first compute the maximum number  $L$  of node-disjoint  $s$ - $t$  paths in  $\hat{\mathcal{G}}$  without considering total node weight (which corresponds to the number of mobile sensors needed) by applying Kumar et al.'s strategy in [17]. Then,  $L$  can serve as the upper bound of the maximum node-disjoint hybrid sensor barrier in our problem.

Now, we perform binary search to find the maximum flow whose cumulative node cost is no greater than  $l$ . For this purpose, we first set  $\alpha \leftarrow \lceil L/2 \rceil$  and run the polynomial time optimal algorithm for the minimum cost  $\alpha$ -flow problem in [33] and obtain flows. Depending on whether the cost of the flow is greater than  $l$  or not, we adjust  $\alpha$  according to the binary search strategy to find maximum achievable  $\alpha$ . Once the search is done,  $\alpha$  and corresponding flow assignment are returned.

*Step 5. Construction of Solution*

The output of the previous step consists of  $\alpha$  node-disjoint  $s$ - $t$  paths such that the total weight of the nodes which are involved in carrying the flows is no greater than  $l$ . Remind that each node  $v$  with node weight is added during Step 2 and the weight of each node presents the minimum number of mobile nodes to connect the two static sensor nodes  $x$  and  $y$  neighboring to  $v$  in  $\hat{\mathcal{G}}$  by Property 1. Therefore, each node-disjoint path can form a single hybrid sensor barrier and with the total available  $l$  mobile sensors, we can construct  $\alpha$  node-disjoint hybrid sensor barriers (Property 2).

Now, we prove our strategy produces an optimal solution within polynomial time.

**Theorem 2.** *The proposed algorithm is an optimal solution for the MFBP-HSN problem.*

*Proof.* Let  $V = \{v_1, v_2, \dots, v_n\}$  be the set of ground nodes. Suppose that  $OPT = \{m_1, m_2, \dots, m_l\}$  is an optimal solution to MFBP-HSN problem (where  $m_i$  are mobile nodes for  $i = 1, 2, \dots, l$ ), and the maximum number of node-disjoint paths from  $s$  and  $t$  is  $f$ . Then we can construct a graph  $\mathcal{G}$  as follows: the vertex-set of  $\mathcal{G}$  is  $V \cup OPT \cup \{s, t\}$  and two nodes  $u$  and  $v$  are adjacent if and only if the distance between them is less than or equal to one. Based on  $\mathcal{G}$ , we can formulate another graph  $H$  as follows: Let  $P_1, P_2, \dots, P_f$  be the maximum node-disjoint paths from  $s$  to  $t$  in  $\mathcal{G}$ . For

every path  $P_i$ , replace the consecutive mobile nodes in  $P_i$  with a virtual node with weight being equal to the number of consecutive mobile nodes, if there are no mobile nodes between two adjacent nodes  $u$  and  $v$  in  $P_i$ , we will add a virtual node with weight 0 in the middle of edge  $(u, v)$ . After the operations above, if there are mobiles in  $\mathcal{G}$ , we simply remove all the rest of them, because they make no contribution in connecting the ground nodes. Then we obtain a graph  $H$ . Finally, we apply Kumar et al.'s idea and convert  $H$  to  $\hat{H}$ . Note in graph  $\hat{H}$ , the minimum-cost-flow from  $s$  to  $t$  is at most  $l$  with flow value  $f$ . Thus, in this way, we reduce the original MFBP-HSN problem to a variant of the minimum-cost-flow problem with budget at most  $l$  in graph  $\hat{H}$ , and the well-known the minimum-cost  $\alpha$ -flow problem [33] suffices for our purpose.  $\square$

**Theorem 3.** *The running time of the proposed algorithm is polynomial.*

*Proof.* Clearly, from Steps 1 to 3, the graphs  $\mathcal{G}$  can be constructed in polynomial time. Step 4 can be done in polynomial time by [17]. Moreover, the running time of the minimum-cost flow algorithm (see e.g. [33]) is polynomial, which will be executed at most  $\log_2 n$  times, and therefore the total running time of Step 5 is also polynomial. This completes the proof.  $\square$

## 6. Concluding Remarks

In this paper, we introduce a new barrier-coverage problem in the hybrid wireless sensor network of both static ground sensor nodes and mobile sensor nodes. Our main contribution of this is along with the new problem, the polynomial time optimal algorithm for the problem of our interest. Thanks to the recent advances in micro-electronic technologies, a wide variety of wireless mobile sensor nodes are introduced in the market and their cost is getting lower. As a result, the construction of efficient hybrid sensor network becomes more feasible. On the other hand, there are generally a lack of understanding on how to use this new mobile nodes to improve the traditional wireless sensor nodes in terms of lifetime, surveillance quality, etc. Therefore, we plan to continue on investigating the applications of mobile sensor nodes in traditional wireless sensor networks, and related optimization problems. We also plan to continue our investigation on the generalized version of the MFBP-HSN problem by eliminating several uniformity and homogeneity assumptions.

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