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A Joint Optimization of Data Ferry Trajectories and Communication Powers of Ground Sensors for Long-term Environmental Monitoring

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Abstract Recently, various hybrid wireless sensor networks which consist of several robotic vehicles and a number of static ground sensors have been investigated. In this kind of system, the main role of the mobile nodes is to deliver the messages produced by the sensor nodes, and naturally their trajectory control becomes a significant issue closely related to the performance of the entire system. Previously, several communication power control strategies such as topology control are investigated to improve energy-efficiency of wireless sensor networks. However, to the best of our knowledge, no communication power control strategy has been investigated in the context of the hybrid wireless sensor networks. This paper introduces a new strategy to utilize the communication power control in multiple data ferry assisted wireless sensor network for long-term environmental monitoring such that the lifetime of the sensor network is maximized. We formally define the problem of our interest and show it is NP-hard. We further prove there exists no approxi-

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mation algorithm for the problem which can produce a feasible solution for every possible problem instance even though there is a feasible solution. Then, we propose heuristic algorithms along with rigorous theoretical performance analysis for both the single data ferry case and the multiple data ferry case under certain condition.

Keywords Wireless sensor network · message ferrying · energy-efficiency · communication power control · path planning · traveling salesman problem

1 Introduction

This paper considers a wireless network of energy-restricted stationary sensor nodes sparsely deployed over a vast remote area, which usually forms a disconnected network. This kind of sensor networks are very useful to collect on-the-ground measurements of isolated regions such as watershed runoff [29] or sea ice mass balance [6], where the sensor nodes have insufficient resources for long-range radio communication and do not have an access to cellular networks. As a result, data ferries, which are fully controllable mobile nodes such as unmanned aerial vehicles (UAVs), are frequently adopted to collect data from the sensor nodes via short range wireless communication and deliver the data to the sink (a designated data collector connected to users) over repeated tours [15]. In those data ferry assisted sensor networks, or equivalently in those hybrid wireless sensor networks, the trajectory control of the data ferries has been an important issue [13–16, 19–23, 26, 28, 30–35].

In the sensor networks for environmental monitoring, each sensor node produces relatively large amount of data per unit period, which is usually not time-sensitive. Therefore, it does no significant harm to deliver the data with some delay. However, since each node has a limited memory capacity, if a sensor node is not visited by a data ferry frequently enough, it may be forced to drop some portion of the accumulated data, which may lead to a significant scientific loss. This means that if a data ferry travels to collect data from the sensor nodes over repeated tours with constant speed, the length of each tour cannot be greater than some limit [7, 11, 12].

Frequently, a limited energy source such as a battery is the only power supply of each sensor node. As a result, energy-efficiency has been a significant issue of wireless sensor networks [10, 25]. The radio frequency (RF) signal is most frequently used for wireless communications in wireless sensor networks. It is well-known that the energy to transmit a radio signal increases super-linearly proportional to the travel distance of the signal. This means that long range communication in wireless sensor networks is energy-exhaustive and will negatively impact the lifetime of the sensor networks. This observation motivated lots of researchers to study various communication power adjustment strategies to extend the lifetime of wireless sensor networks [9]. However, to the best of our knowledge, there has been no effort made to apply this strategy to extend the lifetime of the hybrid wireless sensor networks.

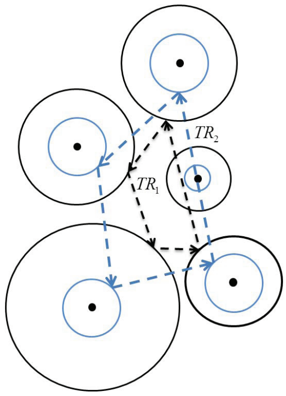

Fig. 1 Given a set of nodes, we can make the length of tour of a data ferry visiting the neighborhood of the nodes shorter by increasing the communication power of the nodes, which will result in shortening the lifetime of the nodes. Still, we may want to increase the communication power sufficiently to have a valid tour of the data ferry which does not allow any buffer overflow. In this figure, a data ferry can use TR_1 (instead of TR_2, which is much longer than TR_1) to collect data from the sensor nodes when the sensor nodes are increasing their communication power (e.g. from smaller circles to larger circles).

This paper studies a new optimization problem to maximize the lifetime of a hybrid wireless sensor network for long-term environmental monitoring by adjusting the communication power of each node and the trajectory of each data ferry without causing any buffer overflow. To the best of our knowledge, this is the *first effort* to understand the fundamental relationship between the communication power of wireless sensor nodes and the lifetime of a wireless sensor network in the context of hybrid wireless sensor networks. The main contribution of this paper is twofold.

(a) **New NP-hard optimization problem**: We introduce a new optimization problem whose goal is, given a hybrid wireless sensor network of k homogeneous data ferries and n homogeneous wireless sensor nodes, to maximize the lifetime of the sensor network by adjusting the communication power of each sensor node and by carefully planning the trajectory of each data ferry such that each sensor node (the neighborhood area of the sensor node) is regularly visited by a data ferry before a buffer overflow occurs (see Fig. 1). We formally define this problem as the *lifetime maximization via communication power adjustment and k data ferry trajectory control (LM-CPkTC)* problem and show it is NP-hard.

(b) **New heuristic algorithms with performance analysis**: we show there exists no approximation algorithm which can produce a feasible solution for every possible LM-CPkTC problem instance even though there exists a feasible solution. Then, we introduce a new heuristic algorithm with theoretical performance guarantee for LM-CP1TC, which is a special case of LM-CPkTC with $k = 1$ under a certain condition. Furthermore, we propose another new heuristic algorithm with theoretical performance guarantee for LM-CPkTC under a specific condition.

The rest of this paper is organized as follows. Related work is presented in Section 2. Section 3 provides the overview of the system model, the formal definition of LM-CPkTC, and the proof of its NP-hardness. Section 4 introduces our main results, the heuristic algorithms with performance guarantee for LM-CPkTC. Lastly, we conclude this paper and provide some future work in Section 5.

2 Related Work

In theory, the problem of computing minimum length tour of a data ferry visiting a group of nodes is known as the traveling salesman problem (TSP), which is NP-hard [1]. Its well-known variation, traveling salesman problem with neighborhood (TSPN), whose goal is to find a minimum length tour to visit the neighborhood areas of a given set of nodes, is also NP-hard. Frequently, TSP and TSPN are used to abstract a single data ferry trajectory computation problem whose goal is to minimize the latency to collect data from a given set of remote sensors [16,26]. Dumitrescu and Mitchell proposed a polynomial time $(\pi+8)(1+\epsilon)$-approximation algorithm for TSPN in which the neighborhood areas of the nodes are uniform circular shaped and not necessarily disjoint, where ϵ is a very small positive integer [4]. They also proposed $(1+\epsilon)$-approximation algorithm for this TSPN in which the neighborhood areas of the nodes are disjoint and arbitrary fat-regions [5]. In [27], Mitchell proposed a constant factor approximation algorithm for TSPN in which the neighborhood areas of the nodes are pairwise disjoint and arbitrary shaped fat-regions. In [17], he also introduced a $(1+\epsilon)$-approximation algorithm for TSPN in which the neighborhood areas of the nodes are not necessarily disjoint and arbitrary fat-regions.

Given a set of nodes and k roots, the k-rooted tree cover problem (k-RTCP) is to find a set of trees each of which spans over a distinct root and a subset of nodes such that each node is visited by a tree and the total edge length of the heaviest tree is minimized. In [8], the authors have proposed a $(4+\epsilon)$-approximation algorithm for k-RTCP as well as another $(4+\epsilon)$-approximation algorithm for k-tree cover problem (k-TCP), which is a variation of k-RTCP without the concept of roots. Very recently, Kim et al. have studied the k-traveling saleman problem with neighborhood (k-TSPN), whose goal is to find k rooted tours spanning over the neighborhood areas of the nodes such that each tour visits a distinct root and the neighborhood area of each node, which is a uniform circular area and not pairwise disjoint with each other, is visited by some tour and the length of the longest tour is minimized [31,34]. This work proposed a constant factor approximation algorithm for k-TSPN, as well as the k-rooted path cover problem with neighborhood (k-PCPN) whose goal is similar to k-TSPN but it is looking for k-paths instead of k-tours.

In [14], the authors showed the energy-efficiency of the wireless sensor networks can be significantly increased by using a data ferry for routing instead of multihop routing. The k-traveling salesman problem (k-TSP) is to find k-tours starting from the same orientation such that the length of the longest tour is minimized. The first constant factor approximation algorithm for k-TSP, namely k-SPLITOUR, is proposed by Frederickson et al [3]. In [20], Tekdas et al. used the approximation algorithm for k-SPLITOUR as a heuristic algorithm to find the tours of k-robots to collect the data from sensor nodes.

In [19], under the assumption that the location of the sensor nodes are not exactly known, the authors show an optimal solution of TSP results in a suboptimal trajectory of data ferry if the goal is minimizing average delay. Under the same assumption, Pearre and Brown have investigated the problem of minimizing the trajectory length of a single data ferry collecting data from widespread stationary sensor nodes [28, 30, 33]. In those works, the data ferry has an initial trajectory to collect the data from the sensor nodes. However, if the data ferry wants to download B bytes of data from each sensor node, depending on the size of B, it needs to adjust the trajectory since the moment that the ferry is connected to a sensor node may not be long enough to download the data. Therefore, the authors have proposed stochastic trajectory computation algorithms for the ferry to optimize its tour iteratively. They assume a line deployment of the sensor nodes.

The authors in [22] introduced a flexible model to calculate the total energy consumed by a sensor node to transfer a message to a fixed trajectory data ferry, which considers the duty cycle of the sensor node. In [13], the authors introduced the Sencar, which is a data ferry whose trajectory is fully controllable, to collect data from sensor nodes. In this paper, the authors tried to maximize the lifetime of the sensor network by flexibly choosing multi-hop routing and the data ferry based routing whose speed and route are controllable. This work showed the significance of an optimized trajectory of a data ferry to accomplish its goal efficiently. A balanced solution between two performance goals, maximizing energy saving and minimizing latency, is sought for a ferry with a fixed route in [23]. Boloni and Turgut have studied the problem of determining whether transmitting a message toward the sink using multi-hop routing path or waiting for a data ferry with a fixed trajectory. A comprehensive survey on energy conservation strategies for wireless sensor networks using data ferry can be found in [24].

To the best of our knowledge, the work by Pearre and Brown in [32] is most closely related to our problem. In their paper, the authors assume the location of sensors are approximately known, and use their technics from [28] to minimize the energy consumption of sensor nodes from radio communication. The main focus of this work is on how to adjust an initially given handcoded trajectory of a data ferry to collect data from widespread sensor nodes in a way that the energy consumption of sensor nodes for radio communication with the data ferry can be reduced. Note that our work in this paper and this work are complementary with each other since our work focuses on the design of initial tours of multiple data ferries.

3 System Model and Problem Definition

3.1 Network and Energy Model

This paper considers a sparse wireless sensor network of widespread n stationary sensor nodes $V = \{v_1, \cdots, v_n\}$ whose initial energy-levels are uniform. For any graph $G = (V, E)$, $V = V(G)$ and $E = E(G)$ are the set of the vertices (representing the sensor nodes) and edges of G, respectively. Consider a sensor node v_i wants to transmit a bit of data to a data ferry f using RF signal. In RF wireless communications, a signal from v_i can be received by f only if the signal to noise ratio of s, SNR_f satisfies

$$SNR_f = \frac{P(v_i, f)}{N + \sum_{v_j \in V \setminus \{v_i\}} P(v_j, f)} \geq T, \tag{1}$$

where $P(v_i, f)$ is the strength of v_i's signal at f, and T is a threshold value. Since we are considering a sparse wireless sensor network, the interference between any two nodes to transmit a message to f at the same time is likely to be negligible. Therefore, we can simplify Eq. (1) as follow after we normalize N and T to 1, respectively:

$$SNR_f = P(v_i, f) \geq 1. \tag{2}$$

We also assume that no two disks each of which is centered at a node and its radius is r_{max} do not overlap/touch with each other. Meanwhile, in radio communications, we have

$$P(v_i, f) = \frac{P_i}{Euc(v_i, f)^\alpha}, \tag{3}$$

where P_i is the communication power of v_i, $Euc(v_i, f)$ is the Euclidean distance between v_i and f, and α is a propagation decay factor, which is dependent on the communication medium. In case of the radio communication in the air, α is roughly 3. From Eq. (2) and Eq. (3), we have

$$P_i \geq Euc(v_i, f)^\alpha. \tag{4}$$

This equation implies that in order to correctly send a message to f, the communication power of v_i should be at least $Euc(v_i, f)^\alpha$.

3.2 Formal Definition of LM-CPkTC

The wireless sensor networks of our interest use RF signals for communications in which each sensor node consumes energy super-linearly proportional to the communication range. It is known that the majority of the energy of a wireless sensor node is consumed for radio communications. Therefore, it is highly beneficial to adjust the communication range of each node properly to maximize the lifetime of the wireless sensor networks. Meanwhile, each sensor node has a limited memory capacity and thus it can store only a limited amount of data. Since each node generates a certain amount of data per time unit, we assume that the node should be visited by a data ferry and flush its memory at least one time per every t time units for some constant t. Under these constraints, the length of any feasible tour for a data ferry is bounded by $L = t \cdot s$, where s is the constant speed of the data ferry. For simplicity, let us assume all of t, s, L are positive integers, which can be achieved by a simple flooring operation, i.e. $s \leftarrow \lfloor s \rfloor$ and $t \leftarrow \lfloor t \rfloor$.

Based on the observations made above, in order to maximize the lifetime of a multiple data ferry assisted wireless sensor network, which can be defined as the time duration from the deployment of the network to the moment when at least one sensor node is exhausted, we need to properly adjust the communication power of each node and determine the trajectory of each data ferry at the same time. The following is the formal definition of this problem.

Definition 1 (LM-CPkTC) Consider a set of n sensor nodes $V = \{v_1, \cdots, v_n\}$ with uniform remaining energy-level $RE \geq 1$ and their adjusted minimum and maximum communication range $r_{min} = 1$ and $r_{max} \geq 1$, respectively. Also, consider a set of k data ferries with a uniform maximum speed s and a tour length upperbound $L = s \cdot t$, where t is the maximum time that a sensor node can accumulate data without overflow caused by the limited memory capacity.

We assume that a sensor node v_i is visited by a data ferry f if the Euclidean distance between the data ferry and the sensor node $Euc(s_i, f)$ is less than or equal to the communication range r_i of the sensor node, or if f is within (or touching) the "neighborhood area" of v_i, which is defined as a disk with radius r_i and centered at v_i. Then, the **lifetime maximization via communication power adjustment and k data ferry trajectory control (LM-CPkTC)** problem is to maximize the lifetime of the sensor network, i.e.

$$\min_{1 \leq i \leq n} \{\lceil \frac{RE}{{r_i}^{\alpha}} \rceil\} \tag{5}$$

is maximized by

(a) adjusting the communication power r_i, such that $r_{min} \leq r_i \leq r_{max}$, of sensor node $v_i \in V$ for each $1 \leq i \leq n$, and
(b) determining the trajectory (tour) U of each data ferry such that the tour length $Len(U)$ is no greater than L and the neighborhood area of each node is visited by some tour.

Note that we assume each mobile node is powerful and directly connected to the sink everywhere. Therefore, once the tour of a mobile node is determined, it repeatedly moves along the tour for an extend time period to relay messages from each node to the sink. We would like to emphasize that we consider a sparse wireless sensor network for environmental monitoring applications in which the communication ranges of the deployed sensor nodes are disjoint. The following theorem shows our problem is NP-hard.

Theorem 1 *LM-CPkTC is NP-hard.*

Proof Consider a subclass of LM-CPkTC with $k = 1$ and $r_{min} = r_{max}$. Then, the decision version of LM-CPkTC in this subclass is to determine if there exists a tour of length at most L spanning over the all of the neighborhood areas of the nodes. However, this is equivalent to a well-known NP-complete problem, the decision version of the traveling salesman problem with neighborhood (TSPN) in Euclidean space [2]. As a result, the subclass of LM-CPkTC is NP-hard and thus this theorem is true.

Note that the goal of LM-CPkTC can be further simplified as minimizing

$$\max_{1 \le i \le n} \{{r_i}^{\alpha}\}. \tag{6}$$

Also, it is noteworthy in case that the nodes are with different remaining energy level, ${r_i}^{\alpha}$ in Eq. (6) has to be multiplied with a coefficient $\frac{1}{e_i}$, which is a function of the remaining energy level of node v_i. That is, with a smaller remaining energy level at v_i, the cost of increasing communication power r_i will increase inversely proportional to the remaining energy level of v_i, which is e_i. In this paper, we limit ourself to the case with $e_i = e_j$ for every node pairs, and leave the case without this assumption as our future work.

3.3 Feasibility Conditions

In this section, we discuss about a special complexity feature of LM-CPkTC. For the simplicity of our discussion, we assume the remaining energy level of each node is uniform and $r_{min} = r_{max} = 1$. Given a single data ferry and a set of sensor nodes, the goal of LM-CP1TC, a special case of LM-CPkTC with $k = 1$, is to adjust the communication power of each sensor node (this is fixed to 1 by our earlier assumption) and determine the tour of the data ferry such that the length of the tour does not exceed a maximum length constraint, L, and the neighborhood areas of all nodes are visited by the tour. This means that if we assign the maximum communication power to all of the nodes, there exists a tour of length at most L visiting all the neighborhood areas as long as the given problem instance is valid. However, if the only possible solution is a tour of length exactly L, then this problem instance is equivalent to computing an optimal solution of the traveling salesman problem with neighborhood (TSPN), which is known to be NP-hard [1]. This means that no suboptimal algorithm can produce a feasible solution of this particular LM-CP1TC problem instance within polynomial time unless $P = NP$.

Since LM-CPkTC is NP-hard, there is no polynomial time exact algorithm for this problem unless $P = NP$. As a result, we study approximation algorithms for LM-CPkTC during the rest of this paper. In most NP-hard optimization problems, an approximation algorithm is valid if it can unconditionally find a feasible solution of the problems as long as it exists. However, this is not the case of LM-CPkTC, and thus LM-CPkTC is theoretically very

Algorithm 1 BiSec-Adjuster $(V, r_{min}, r_{max}, L, \epsilon)$ /* inputs */

Apply Christofides's 1.5-approximation algorithm for TSP to V and obtain a center tour U.
$r_{low} \leftarrow r_{min}$ and $r_{high} \leftarrow r_{max}$.
loop
  $U' \leftarrow$ NeigTour(U, r_{high}).
  if $Len(U') > L$ **then**
    if $r_{high} = r_{max}$ **then**
      Return $\langle failure, null, null \rangle$. // no solution.
    else
      $r_{low} \leftarrow r_{high}$ and $r_{high} \leftarrow r_{oldhigh}$.
    end if
  else
    if $r_{high} - r_{low} \leq \epsilon$ **then**
      break; /* quit this loop. */
    end if
    $r_{oldhigh} \leftarrow r_{high}$ and $r_{high} \leftarrow \frac{r_{high} - r_{low}}{2}$.
  end if
end loop
Return $\langle success, U_{out}, r_{high} \rangle$, where U_{out} is the most recently computed U' such that $Len(U_{out}) \leq L$. /* outputs */

challenging to deal with. In the following section, therefore, we propose heuristic algorithms for LM-CPkTC with performance guarantee under different assumptions and specify their feasibility condition.

4 Main Results: Approximation Algorithms for LM-CPkTC under Different Assumptions

4.1 Approximating LM-CPkTC with $k = 1$

In this section, we study a special case of LM-CPkTC with $k = 1$. Let us call this simplest case as SLM-CP1TC (Simplified LM-CP1TC). Algorithm 1 is the description of our algorithm for SLM-CP1TC, namely BiSec-Adjuster. Note that the goal of SLM-CP1TC can be restated as "finding the minimum uniform communication power of all nodes in V such that there exists a tour U over the neighborhood areas of all nodes in V such that its length $Len(U)$ does not exceed a limit L," because if we have a solution in which the communication power of the nodes are not uniform, the node with highest communication power will be exhausted faster, and we can potentially decelerate this by increasing the communication power of the other nodes with lower communication power. That is, non uniform adjustment of the communication power does no good.

The algorithm uses Christofides' 1.5-approximation algorithm [1] for TSP and computes a center tour U of all nodes in V. Also, it sets $r_{low} \leftarrow r_{min}$ and $r_{high} \leftarrow r_{max}$. In the following, we will perform a binary search to adjust r_{low} and r_{high} such that

(a) the length of a neighborhood tour of V by our subroutine NEIGTOUR with communication power r_{high} is no greater than L
(b) communication range $r \in [r_{low}, r_{high}]$, and
(c) $r_{high} - r_{low} < \epsilon$, where ϵ is the precision factor, which is very small positive constant.

Now, we discuss about some details. We first discuss about the subroutine NEIGTOUR. Then, we discuss about the main strategy of the algorithm.
NEIGTOUR. In this algorithm, NEIGTOUR is designed to compute a neighborhood tour U' from a center tour U and the communication radius of each node r such that as the radius decreases, the length of resulting tour $Len(U')$ increases monotonically (see Fig. 2). For this purpose, the algorithm first identifies two points, $p_{i,1}$ and $p_{i,2}$, where the border of the disk-shaped neighborhood area with radius r of each node $v_i \in V$ and the center tour U intersect. Then, for each v_i, there exists always two arcs on the border of the neighborhood disk of v_i. Suppose A is the arc heading toward the inside area of U. Suppose cp_i is the center point between $p_{i,1}$ and $p_{i,2}$ on A. Finally, generate a tour U' of $cp_1, cp_2, \cdots, cp_n, cp_1$. Note that the length of this tour is shorter than U. Also, it is easy to see that as the communication radius decreases, the tour length increases.
Main Strategy. Given that r_{high} is enough communication radius for our subroutine NEIGTOUR to find a neighborhood tour whose length is no greater than L, NEIGTOUR may find another tour with smaller communication radius r_{new}. Then, if such a r_{new} exists, $r_{new} \in [r_{low}, \frac{r_{high}-r_{low}}{2}]$. To check if this is correct, we use $\frac{r_{high}-r_{low}}{2}$ as new communication radius and apply NEIGTOUR. If the algorithm outputs a tour whose length is no greater than L, then it is correct. Otherwise, this is not correct, which means that we have to search $[\frac{r_{high}-r_{low}}{2}, r_{high}]$ to further optimize the communication range. Note that this is a kind of binary search strategy, whose running time is logarithm and thus will work really fast. By performing this repeatedly, we will eventually make the difference between r_{high} and r_{low} no greater than ϵ. Once this happens, the algorithm outputs current r_{high} as the communication power which is at most ϵ larger than the optimal communication power, i.e. $r_{opt} + \epsilon \geq r_{high}$, and therefore ϵ becomes the error bound.

Now, we show Algorithm 1 is a constant factor approximation algorithm of SLM-CP1TC under certain conditions and also provides its feasibility condition.

Lemma 1 $1 - \tan\theta \geq \frac{\pi}{4} - \theta$ *for* $0 < \theta < \pi/4$.

Proof Note that $\tan(\pi/4) = 1$. It follows from the Mean Value Theorem that there exists a ξ with $\theta < \xi < \pi/4$ such that

$$\frac{1 - \tan\theta}{\pi/4 - \theta} = \frac{1}{\cos^2\xi} \geq 1.$$

Theorem 2 *Let* L_C *be the length of the center tour* U *computed by Line 1 of Algorithm 1, and* L_A *be the length of the tour* U' *returned by Algorithm 1.*

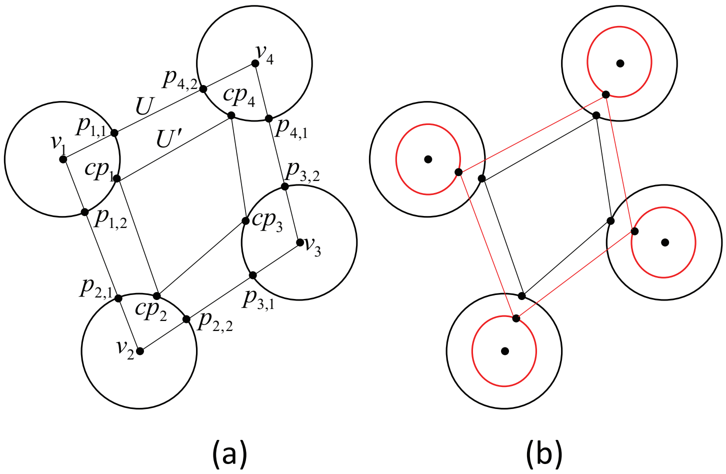

Fig. 2 Fig (a) shows how NEIGTOUR outputs U' from U. Fig (b) shows as the uniform communication power of sensor nodes decreases, the length of the output tour of NEIGTOUR increases monotonically.

Suppose that the center tour U is convex. Then We have $L_A \leq L_C - \pi r$, where r is the transmission range of the sensors.

Proof Let $A_1 A_2 \cdots A_n A_1$ be the order of the nodes visited by the tour U which is a n-polygon (see Fig. 3(a)), and

$$L_C = \sum_{i=1}^{n} |A_i A_{(i \mod n)+1}|.$$

Let $A'_1 A'_2 \cdots A'_n A'_1$ be the corresponding modified tour obtained by Algorithm 1 (essentially by NEIGTOUR) and

$$L_A = \sum_{i=1}^{n} |A'_i A'_{(i \mod n)+1}|.$$

Suppose that $B_1 B_2 \cdots B_n$ is a n-polygon such that $B_i B_{(i \mod n)+1}$ is tangent with the circle $\odot C_i$ and $\odot C_{(i \mod n)+1}$ centered at A_i and $A_{(i \mod n)+1}$ with radius r, respectively. Clearly $A_i B_i$ intersects the circle C_i at A'_i. Let $L' = \sum_{i=1}^{n} |B_i B_{(i \mod n)+1}| + 2 \sum_{i=1}^{n} |A'_i B_i|$. First we show that $L_A \leq L'$. Actually, by triangle inequality, we have

$$|A'_i A'_{(i \mod n)+1}| \leq |A'_i B_i| + |B_i B_{(i \mod n)+1}| + |A'_{(i \mod n)+1} B'_{i+1}|. \tag{7}$$

Summing up he above inequalities for $i = 1, 2, n \cdots, n$ gives the desired inequality.

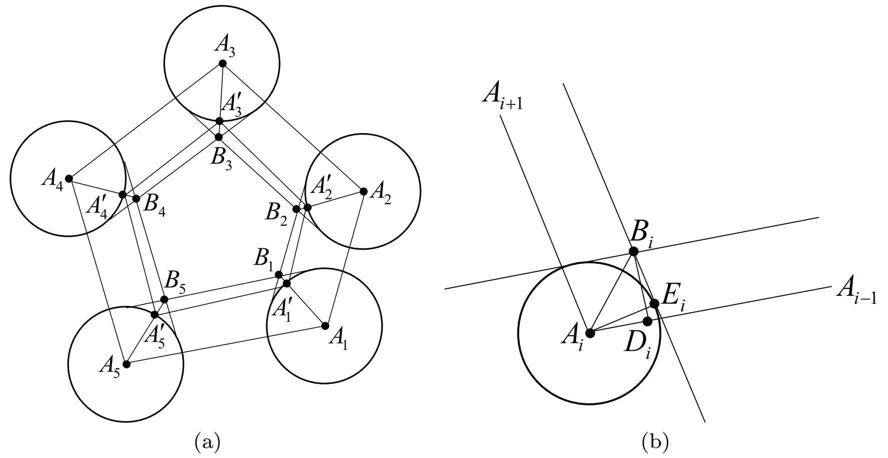

Fig. 3 The illustration of the notations in Theorem 2.

Let B_iD_i be the line segment that is perpendicular with $A_iA_{(i \mod n)+1}$ with D_i as the perpendicular foot. Let A_iE_i be the radius of $\odot C_i$ that is perpendicular with $B_{(i-1) \mod n} iB_i$ with E_i as the perpendicular foot (see Fig. 3(b)). Let $\measuredangle A_{(i-1) \mod n}A_iA_{(i \mod n)+1} = \theta_i$. From triangle $A_iB_iE_i$ we know that $|A_iB_i| = \frac{r}{\sin(\theta_i/2)}$ (since $\measuredangle A_iB_iE_i = \measuredangle B_iA_iD_i = \theta_i/2$); and

$$|A_iD_i| = |A_iB_i|\cos(\theta_i/2) = r\cot(\theta_i/2).$$

It is clear that $L' = L_C - \sum_{i=1}^n 2|A_iD_i| + 2\sum_{i=1}^n |A_i'B_i|$. Note that $|A_i'B_i| = |A_iB_i| - r$. It follows from Lemma 1 that

$$\begin{aligned} L_A \le L' &= L_C - \sum_{i=1}^n 2r(\cot(\theta_i/2) - \frac{1}{\sin(\theta_i/2)} + 1) \\ &= L_C - 2r\sum_{i=1}^n (1 - \tan(\theta_i/4)) \\ &\le L_C - 2r\sum_{i=1}^n (\frac{\pi}{4} - \frac{\theta_i}{4}) \\ &= L_C - \pi r. \end{aligned}$$

The last equality follows from the fact that $\sum_{i=1}^n \theta_i = (n-2)\pi$.

Theorem 3 *Let L_C be the length of the center tour U computed by Line 1 of Algorithm 1, and L_A be the length of the tour U' returned by Algorithm 1. Then We have $L_A \ge \frac{L_C}{1+8/\pi} - \frac{8}{1+8/\pi}r$, where r is any transmission range of the sensors.*

Proof By triangle inequality, we have $L_C \le L_A + 2nr$. Using a similar argument introduced by Dumitrescu and Mitchell [4], we have $\pi r^2 n \le 4lr + 4\pi r^2$, where

l is the optimal solution of of TSPN with radius r. It follows that $n \leq 4 + \frac{4l}{\pi r}$. Thus, $L_C \leq L_A + 2r(4 + \frac{4l}{\pi r}) \leq L_A + \frac{8l}{\pi r} + 8r$. Note $l \leq L_A$, we get $L_C \leq (1 + \frac{8}{\pi})L_A + 8r$.

Lemma 2 *$L_A \leq (1+\frac{8}{\pi})L_{opt}+(8-\pi)r$, where L_{opt} is the length of the optimal tour for TSPN with radius r.*

Proof By using arguments similar to [4], we have $L_C \leq (1 + \frac{8}{\pi})L_{opt} + 8r$. By Theorem 2, we have $L_A \leq L_C - \pi r \leq (1 + \frac{8}{\pi})L_{opt} + (8 - \pi)r$.

Theorem 4 *Suppose that the center tour U is convex. For any input satisfying $L_C/L > (1 + \frac{8}{\pi})^2 + \epsilon_0$, Algorithm 1 is a constant approximation algorithm, where ϵ_0 is a constant.*

Proof Let r_1 be the output of algorithm 1 with $Len(r_1) = L$. Let r^* be the optimal transmission range for our problem. We show that r_1/r^* is upper bounded by some constant under appropriate conditions.

Let r_2 be the transmission range such that $Len(r_2) = (8 - \pi)r_2 + (1 + 8/\pi)L$, we show that $r^* \geq r_2$. (r_2 must exist, consider the function $f(r) = Len(r) - ((8 - \pi)r + (1 + 8/\pi)L)$. When $r = 0, f(0) = L_C - (1 + \pi/8)L > 0$, and $f(r_{max}) < 0$ since $Len(r_{max}) \leq L$. Then the assertion follows from the intermediate Value Theorem for continuous functions). Actually, by above lemma, we have $Len_{opt}(r_2) \geq \frac{Len(r_2)-(8-\pi)r_2}{1+8/\pi} = L$, where $Len_{opt}(r_2)$ is the length of the optimal solution for TSPN with radius r_2. Thus, the assertion has to be true.

Next, we show that r_1/r_2 is upper bounded. By Theorem 2, we have $r_1 \leq \frac{L_C-L_A}{\pi}$. By Theorem 3, we have $Len(r_2) \geq \frac{L_C}{1+8/\pi} - \frac{8r_2}{1+8/\pi}$. It follows that $Len(r_2) = (8 - \pi)r_2 + (1 + 8/\pi)L \geq \frac{L_C}{1+8/\pi} - \frac{8r_2}{1+8/\pi}$. Thus we get $r_2 \geq \frac{L_C-(1+8/\pi)^2 L}{8+(8-\pi)(1+8/\pi)}$. It follows that $\frac{r_1}{r_2} \leq \frac{8+(8-\pi)(1+8/\pi)}{\pi} \frac{L_C/L}{L_C/L-(1+8/\pi)^2} \leq \frac{8+(8-\pi)(1+8/\pi)}{\pi}(1 + (1 + 8/\pi)^2/\epsilon_0)$.

Feasibility Condition of SLM-CP1TC. Feasibility condition: If $Len(r_{max}) > L$, Algorithm 1 fails; otherwise it outputs a feasible solution (i.e., a tour with length $\leq L$ that visits all neighborhood of the sensors with an appropriate radius). If the ratio of L_C (which measures the actually tour length if the transmission range is set to be zero) and L is larger than a threshold value of $(1 + 8/\pi)^2 = 12.58\cdots$, Algorithm 1 guarantees to output a solution with transmission range (radius) no more than the optimal one by a multiplicative constant factor. This is intuitively meaningful , since if L_C/L is relatively large, then the benefit for increasing the radius is apparent.

4.2 Approximating LM-CPkTC with $k \geq 2$

In this section, we study a special case of LM-CPkTC in which the remaining energy level of the sensor nodes is uniform, but k is an arbitrary positive

Algorithm 2 k**-BiSec-Adjuster** $(V, r_{min}, r_{max}, L, \epsilon)$ /* inputs */

Apply the $(4+\epsilon)$-approximation algorithm for k-TCP in [8] over $\langle V, k\rangle$ and obtain a set T of k center trees $\{T_1, T_2, \cdots, T_k\}$.
for each i such that $1 \le i \le k$ **do**
  $\langle RT_i, U_i, r_i\rangle \leftarrow$
    BiSec-Adjuster$(V(T_i), r_{min}, r_{max}, L, \epsilon)$.
end for
for each i such that $1 \le i \le k$ **do**
  if $RT_i = failure$ **then**
    Return $\langle failure, null, null\rangle$.
  end if
end for
Return $\langle success, \{U_1, \cdots, U_k\}, \{r_1, \cdots, r_k\}\rangle$. /* outputs */

integer, i.e. $k \ge 1$. Let us call this simplest case as SLM-CPkTC (Simplified LM-CPkTC). Let us introduce some preliminaries first.

Definition 2 (MSTN) Given a set V, the *minimum spanning tree with neighborhood (MSTN)* of V is a spanning tree with smallest total length spanning over the neighborhood areas of all nodes in V, which are not touching or overlapping with each other.

Lemma 3 *[18] Given a set V of nodes $\{v_1, v_2, \cdots, v_n\}$, let T^{center}_{mst-I} and T^{disk}_{mst-I} be an MST of V and an MST of a set of "non-overlapping" disks $\{N(v_1), \cdots, N(v_n)\}$ with radius 1. Then, $Len(T^{center}_{mst-I}) \le (1 + 20/\pi)Len(T^{disk}_{mst-I}) + 2r$, where r is the uniform radius of the disks.*

Definition 3 (k-TCP) Given a set V of n nodes and a positive integer k, the *k-tree cover problem (k-TCP)* is to find a set of k trees $T = \{T_1, T_2, \cdots, T_k\}$ such that

(a) for each $v_j \in V$, $\exists T_i \in T$ visiting v_j, i.e. $v_j \in V(T_i)$, and
(b) $Cost(T) = \max_{1 \le i \le k} Len(T_i)$ is minimized.

Theorem 5 *[8] There is a $(4+\epsilon)$-approximation algorithm for k-TCP.*

Definition 4 (k-TCPN) Given a set V of n nodes and a positive integer k, the *k-tree cover problem with neighborhood (k-TCPN)* is to find a set of k rooted-trees $T = \{T_1, T_2, \cdots, T_k\}$ such that

(a) for each $v_j \in V$, $\exists u \in V(T_i)$ for some $T_i \in T$ such that u is in (or on the border of) $N(v_j)$, and
(b) $Cost(T) = \max_{1 \le i \le k} Len(T_i)$ is minimized.

Definition 5 (k-TSPN) Given a set V of n nodes and a positive integer k, the *k-traveling salesman problem with neighborhood (k-TSPN)* is to find a set of k tours $U = \{U_1, U_2, \cdots, U_k\}$ such that

(a) for each $v_j \in V$, $\exists u \in V(U_i)$ for some $U_i \in U$ such that u is in (or on the border of) $N(v_j)$, and

(b) $Cost(U) = \max_{1\le i\le k} Len(U_i)$ is minimized.

Algorithm 2 is our approximation algorithm for SLM-CPkTC. This algorithm is similar to Algorithm 1. However, since we have k different data ferries, it first applies an existing algorithm for k-TCP in [8] over $\langle V, k\rangle$ and partition the nodes into k subgroups (Line 3). Then, for each subgroup, the strategy in Algorithm 2 is applied to determine the communication power of each node and the tour of each data ferry. Now, we introduce the following key lemma which will be used to prove the performance ratio of Algorithm 2.

Lemma 4 *Let $L^{(j)}_{opt}$ be the length of the optimal solution for TSPN on the subset $V(T_j)$, and L_{opt} the cost of an optimal solution of k-TSPN on the set V, both with radius r. Then $L^{(j)}_{opt} \le \gamma_1 L_{opt} + \gamma_2 r$, for some constants γ_1 and γ_2.*

Proof Let r be the uniform communication range of each node. Suppose we have a set V of n nodes such that the neighborhood areas of any pair of node are disjoint (and not touching) and a set O of k mobile nodes. Now, consider we obtain a set U' of k tours $\{U'_1, U'_2, \cdots, U'_k\}$ after the execution of Line 4 of Algorithm 1 (which used as a sub-procedure of Algorithm 2). Also, suppose $U^{OPT-N} = \{U^{OPT-N}_1, U^{OPT-N}_2, \cdots, U^{OPT-N}_k\}$ is an optimal solution of k-TSPN problem instance defined over $\langle V, O\rangle$. Now, we will show that

$$Len(U') \le 1.5\big(4 + \epsilon_1\big)[(1 + 20/\pi)Len(U^{OPT-N}) + 2r], \qquad (8)$$

where ϵ_1 is a very small independent constant. Then, this lemma is true, since

$$L^{(j)}_{opt} \le Len(U') = \max_{1\le i\le k} Len(U'_i)$$

for any j. Let $T^C = \{T^C_1, T^C_2, \cdots, T^C_k\}$ be the set of the k trees connecting the nodes in V computed in Line 1 of Algorithm 2 (this part is applied to each of T_i computed by Line 1 of Algorithm 2). Then, by Theorem 5, we have

$$Len(T^C) = \max_{T^C_i \in T^C} Len(T^C_i) \le (4 + \epsilon_1)Len(T^{OPT-C}), \qquad (9)$$

where $T^{OPT-C} = \{T^{OPT-C}_1, T^{OPT-C}_2, \cdots, T^{OPT-C}_k\}$ is an optimal solution of k-TCP defined over $\langle V, O\rangle$, and

$$Len(T^{OPT-C}) = \max_{T^{OPT-C}_i \in T^{OPT-C}} Len(T^{OPT-C}_i).$$

By Lemma 3, we have

$$Len(T^{OPT-C}) \le (1 + 20/\pi)Len(T^{OPT-N}) + 2 \cdot r. \qquad (10)$$

where $T^{OPT-N} = \{T^{OPT-N}_1, T^{OPT-N}_2, \cdots, T^{OPT-N}_k\}$ is an optimal solution of k-TCPN defined over $\langle V, O\rangle$ with radius r.

Furthermore, given a set of points, the cost of a spanning tree is always bounded by the cost of a tour, we have

$$Len(T^{OPT-N}) \leq Len(U^{OPT-N}). \tag{11}$$

By combining Eq. (9), Eq. (10), and Eq. (11), we have

$$Len(T^C) \leq (4+\epsilon_1)\Big[(1+20/\pi)Len(U^{OPT-N}) + 2r]. \tag{12}$$

Through NEIGTOUR in Line 4 of Algorithm 1, we convert the nodes in each tree $T_i^C \in T^C$ into a tour connecting the neighborhood areas of the nodes in $V(T_i^C)$. However, this neighborhood tour, say U_i' does not increase its size from $1.5Len(T_i^C)$. As a result, we have

$$\begin{aligned} &Len(U') = \max_{1\leq i\leq k} Len(U_i') \\ &\leq 1.5\big(4+\epsilon_1\big)[(1+20/\pi)Len(U^{OPT-N}) + 2r], \end{aligned}$$

and thus this lemma is true.

Theorem 6 *Suppose that the center tour U_i is convex. For any input satisfying $L_C^{(i)}/L > \gamma_1(1+8/\pi)^2 + \epsilon_0$ (ϵ_0 is a constant, $i = 1, 2, \cdots, k$), Algorithm 2 is a constant factor approximation algorithm.*

Proof The performance of Algorithm 2 is determined by the largest transmission range among the k tours output by Algorithm 2. Suppose $r_A^{(i)}$ is the maximum radius which is attained by the i-th sub tour, and r^* is the optimal one, we have to show $r_A^{(i)}/r^*$ is upper-bounded.

Similar to the proof of Theorem 4, our strategy is to find some intermediate radius r_B such that r_B is a lower-bound of r^*, then try to show that $r_A^{(i)}/r_B$ is the upper bounded.

By Theorem 2, we have $L_A^{(i)} \leq L_C^{(i)} - \pi r_A^{(i)}$. It follow that $r_A^{(i)} \leq \frac{L_C^{(i)} - L_A^{(i)}}{\pi}$.

Note that Theorems 2 and 3, and Lemma 2 can still be applied to the i-th subtour. Thus, we have

$$L_A^{(i)} \leq L_C^{(i)} - \pi r_A^{(i)}. \tag{13}$$

$$L_A^{(i)} \geq \frac{L_C^{(i)}}{1+8/\pi} - \frac{8}{1+8/\pi} r_A^{(i)}. \tag{14}$$

$$L_A^{(i)} \leq (1+8/\pi)L_{opt}^{(i)} + (8-\pi)r_A^{(i)}, \tag{15}$$

where $L_{opt}^{(i)}$ is the length of the optimal solution on the subset of $V_i = V(T_i)$.

Also, by Lemma 4, we have $L_{opt}^i \leq \gamma_1 L_{opt} + \gamma_2 r$, where L_{opt} is the length of the optimal solution of k-TSPN with radius r, γ_1 and γ_2 are two absolute constants.

Using the similar arguments as in the proof of Theorem 4, we have

$$\frac{L_C^{(i)}}{1+8/\pi} - \frac{8}{1+8/\pi} r_B \leq L_A^{(i)} \leq (1+8/\pi)L_{opt}^{(i)} + (8-\pi)r_B.$$

It follows that

$$L_{opt}(r_B) \geq \frac{L_C^{(i)} - (8 + \gamma_1(1 + 8/\pi) r_B}{\gamma_2 (1 + 8/\pi)^2},$$

i.e.,

$$L_C^{(i)} - 8r_B \leq (1 + 8/\pi)^2 (\gamma_1 L_{opt} + \gamma_2 r_B) + (8 - \pi) r_B.$$

Choose r_B such that

$$\frac{L_C^{(i)} - (8 + (8 - \pi)(1 + 8/\pi) + \gamma_2 (1 + 8/\pi)^2) r_B}{\gamma_1 (1 + 8/\pi)^2} = L. \tag{16}$$

Then it is clear that $r^* \geq r_B$. Next, we show that $r_A^{(i)}/r_B$ can be upper bounded.

It follows from Eq. (16) that

$$\frac{r_A^{(i)}}{r_B} \leq \frac{L_C^i - L_A^i}{\pi} \Big/ \frac{L_C^{(i)} - \gamma_1 (1 + 8/\pi)^2 L}{8 + (8 - \pi)(1 + 8/\pi) + \gamma_2 (1 + 8/\pi)^2}.$$

i.e.,

$$\begin{aligned} \frac{r_A^{(i)}}{r_B} &\leq \frac{8 + (8 - \pi)(1 + 8/\pi) + \gamma_2 (1 + 8/\pi)^2}{\pi} \frac{L_C^i / L}{L_C^i / L - \gamma_1 (1 + 8/\pi)^2} \\ &\leq \frac{8 + (8 - \pi)(1 + 8/\pi) + \gamma_2 (1 + 8/\pi)^2}{\pi} (1 + \gamma_1 (1 + 8/\pi)^2 / \epsilon_0). \end{aligned}$$

The left hand side is upper-bounded by a constant by the assumption, thus the theorem holds. This complete the proof.

Feasibility Condition of Algorithm 2. Algorithm 2 guarantees to produce a feasible solution as long as $Len(U^{(0)}) \leq L$. This is because, if the length of the initial tour, $U^{(0)}$ computed by the PTAS for TSPN, is greater than L, then this algorithm fails. Clearly, this is possible to happen. For instance, if the length of shortest tour is exactly L, the PTAS may generate a tour of length $(1 + \epsilon)L$ which is greater than L for some positive constant ϵ.

5 Conclusion

In this paper, we introduced a new strategy to exploit communication power adjustment strategy to maximize the lifetime of hybrid wireless sensor networks for long-term environmental monitoring applications. Under the uniformness assumptions on both sensor nodes (e.g. equivalent minimum and maximum communication power, uniform buffer size, uniform remaining energy-level, etc.) and data ferries (i.e. equivalent speed), we obtain constant factor approximation algorithms for the case with one data ferry as well as the case with multiple data ferries under a certain condition. We believe this paper will serve as a seminary work for various interesting research directions. In particular, the problem of our interest assume the remaining energy level of each node pair are same. As we discussed earlier, the investigation of the case without this assumption will require to modify Eq. 6 first, and then reconsider the algorithms and performance analysis of the rest of this paper. Meanwhile, this paper models each data ferry are fixed to a computed route and would not interfere or help each other. However, it is possible that a data ferry which finished its task earlier may help the other ferry to finish their task earlier. This new strategy may possibly help to improve the performance of the whole system, but also require to solve a very tough geo-topological scheduling problem along with multiple data ferries. We plan to investigate this case in our future researches.

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