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Improving the influence under IC-N model in social networks

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The influence maximization problem in social networks is to find a set of seed nodes such that the total influence effect is maximized under certain cascade models. In this paper, we propose a novel task of improving influence, which is to find strategies to allocate the investment budget under IC-N model. We prove that our influence improving problem is $\mathcal{NP}$-hard, and propose new algorithms under IC-N model. To the best of our knowledge, our work is the first one that studies influence improving problem under bounded budget when negative opinions emerge. Finally, we implement extensive experiments over a large data collection obtained from real-world social networks, and evaluate the performance of our approach.

Keywords: Improving influence; bounded budget; social networks.

Mathematics Subject Classification 2010: 05C85, 68W25

1. Introduction

With the rapid development of information technology, the connection between the people are getting more and more frequent. One of the most important platforms is social networks, such as Facebook, Twitter, Flicker, etc., which are large networks that represent the connections among people. Social networks are ubiquitous in the real world. People in a social network share information (opinions, news, etc.), get

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benefit, propagate their opinions (positive or negative) and influence their related individuals like their family, friends and so on, and the people who receive others' opinions may make a decision according to the opinions. Moreover, the opinions can propagate through the network to more people, and many individuals will be influenced at last. This motivates numerous researchers to conduct extensive studies on social networks. One of the fundamental problems is influence maximization, which was firstly introduced by Richardson and Domingos [7, 17] as an algorithmic problem.

Influence maximization problem is to select a seed set containing k elements such that its total influence effect is maximized. Kempe *et al.* in [12] further studied this problem and expanded it to the directed social networks, they formulated influence maximization problem into an optimization problem on the basis of two classical propagation models: the *Independent Cascade* (IC) Model and *Linear Threshold* (LT) Model.

IC model: Each edge $(u; v)$ in the graph is associated with a propagation probability $\mathrm{pp}(u; v)$, which is the probability that node u independently activates (also known as influences) node v at step $t + 1$ if u is activated at step t. In [3], Chen *et al.* proposed Independent Cascade model with Negative opinions (IC-N) on the basis of IC model, which includes positive opinions and negative opinions. In IC-N the negative opinions may propagate and affect many people's decisions as well as the positive opinions, and this model is closer to the real world. Based on the IC-N model, when the seeds are selected, the cost of a seed node may be very high, the problem of how to select some nodes in order to improve the positive influence under a bounded budget B deserves study. We should try to improve the probability of positive opinions spreading in the network.

To the best of our knowledge, our paper is the first one that studies improving the positive influence under certain budget in social networks. To summarize, this paper has the following contributions.

(1) We define and formulate the problem of improving influence in social networks. We transform the problem to how to effectively increase the probability of positive opinions of some nodes such that the expected positive influence in the social network is maximized;
(2) We analyze the problem and prove the $\mathcal{NP}$-hardness by reduction from the knapsack problem, we compute the influence in tree structure, and give efficient algorithms to solve the problem in IC-N model under the bounded budget;
(3) We evaluate our proposed algorithm on real world networks. Experimental results show that our algorithm is efficient and effective.

The rest of this paper will be organized as follows. The related work will be reviewed in Sec. 2. We will define our problem and formulate it in Sec. 3. In Sec. 4, the problem of improving positive influence will be described and analyzed, moreover, the $\mathcal{NP}$-hardness of the problem will be proved in this section. In Sec. 5, we will show the experimental settings and results. Finally in Sec. 6 we will present the future research and the conclusion of this paper.

2. Related Works

Influence maximization is first studied as an algorithmic problem in [7, 17]. However, Kempe *et al.* [12] were the first to formulate the problem as a discrete optimization problem. They showed that the problem is NP-hard under both IC and LT models, and proposed the hill-climbing algorithm, which provide approximation guarantees arbitrarily close to $(1 - 1/e)$. And then a number of works focus on influence maximization for IC model [5, 11, 18, 15, 4, 9]. Leskovec *et al.* [14] improved the hill-climbing algorithm, and proposed the CELF optimization in order to reduce the number of influence spread evaluations. Chen *et al.* [6] proposed new greedy and degree discount heuristics algorithms. Their algorithms are faster than the CELF optimized algorithm, which makes their influence spread close to the greedy algorithm. In [10], Goyal *et al.* proposed CELF++, an extension to CELF. They reduced the cost of recomputing the marginal gain in any iteration, and showed that CELF++ is approximately 35–55% faster than CELF. Chen *et al.* [5] proposed a Maximum Influence Arborescence (MIA) heuristic algorithm using local arborescence structure of each node to approximate the influence propagation for the general IC model. The authors show that the influence spread in MIA model is submodular and the algorithm guarantees an influence spread within $(1 - 1/e)$ of the optimal solution. Jiang *et al.* [11] proposed a different approach based on Simulated Annealing (SA) for the influence maximization problem, two heuristic methods are given: one to accelerate the convergence process of SA and the other of computing influence to speed up the proposed algorithm. In [19], Zhu *et al.* designed a semidefinite programming based algorithm that can give performance ratio better than $(1 - 1/e)$ under some certain cases.

There are some further researches in the case where negative opinions may emerge based on the existing characters of social networks, Chen *et al.* in [3] proposed a new extension to the IC-N model, and gave its properties. They proposed an algorithm to compute the influence in tree structures, and proved a greedy algorithm with $(1-1/e)$ ratio. There are also some other studies assumed that negative impact is usually stronger and more dominant than positive impact in the process of people make decisions [8, 1, 13, 16, 2]. These literatures limit the negative opinions from spreading in social networks.

3. Problem Formulation

In this section, at first we will present some notations which will be used in the paper, and then we will give the related definitions and formulate the problem description of improving influence under IC-N model. Finally we will prove the $\mathcal{NP}$-hardness of the problem.

Consider a social network as a directed graph $G = (V, E, p, w, q)$, where each node $v_i \in V$ represents an individual in the network, and an edge $(v_i, v_j) \in E$ means that individual v_i has a relation with v_j, the weight $p(v_i, v_j)$ denotes the influence probability of v_i on v_j, $w(v_i)$ represents the weight of individual v_i, when v_i is influenced by a positive neighbor, it has the possibility of $q(v_i)$ to turn positive, and the possibility of $1 - q(v_i)$ to turn negative. In this paper, when v_j is chosen, its weight $w(v_j)$ is added into the budget. If we allocate $w(v_j)$ budget to v_j, which is equivalent to select v_j once, the increase function of influence on edges is $\Delta q(v_j) = \frac{q(v_j)}{|N^{\text{in}}(v_j)|}$ according to degree and community structure. All the notations we use in this paper are summarized in Table 1.

We give the definition of our problem as follows.

Definition 1 ($\mathbf{I^2}$NOE problem). Improving Influence when Negative Opinions Emerge. Given $G = (V, E, p, w, q)$ and a budget B, the problem is to select some individuals beyond seeds such that the positive influence of the seed set A on G can be improved as much as possible under budget B. The nodes can be selected more than one times.

Based on the above definition, we now formally formulate the $\mathrm{I^2}$NOE problem in IC-N model.

3.1. *Independent cascade when negative opinions emerge (IC-N model)*

According to the previous research works and definitions, the total influence of TC generated from seed set A can be formulated as follows:

$$\sigma(A) = \sum_{G' \in \Gamma_G} \mathrm{Prob}(G')\sigma_{G'}(A), \tag{1}$$

where $\sigma(A)$ is the total influence of TC generated from seed set A, and $G' = (V', E', p')$ is a subgraph of G, where $V' = V$, $E' \subseteq E$, and $p'(v_i, v_j) = p(v_i, v_j)$, Γ_G denotes the set of all such subgraphs G'. $\sigma_{G'}(A)$ is the number of nodes that

Table 1. Notation List.

Notations	Descriptions
n	Size of the social network
A	Seed set
$N^{\text{in}}(v)$	Set of in-neighbors of v
$w(v_i)$	The weight of v_i
$\inf(v_i)$	Value of A positive impact on v_i
q_i	The original probability of positive opinion of v_i
q'_i	The probability of positive opinion of v_i after improved
$\sigma(A)$	Total influence of a graph G
$h(A, v)$	Hop count of shortest path from A to v
$\mathcal{P}(A, v)$	All the paths from A to v with the hop is $h(A, v)$

can be reached by live-edge path from any node in A and all nodes on the live-edge path are positive.

From now on, for the sake of simplicity, we abbreviate $q(v_j)$ and $w(v_j)$ as q_j and w_j, respectively. Through [12], G' is obtained with probability

$$\mathrm{Prob}(G') = \prod_{(v_i,v_j)\in E'} p(v_i, v_j), \tag{2}$$

$$\sigma_{G'}(A) = \sum_{i=0}^{n-1} a_{G'}(A, i) \prod_{j=0}^{i} q_j, \tag{3}$$

where $a_{G'}(A, i)$ is the number of nodes that are i hops away from seed set A in G', i.e., $a_{G'} = |\{v \mid d_{G'}(A, v) = i\}|$, which is mentioned in [3].

According to the formula (1), q_i will be changed to q'_i after node i is allocated a budget:

$$\sigma_{G'}(A) = \sum_{i=0}^{n-1} a_{G'}(A, i) \prod_{j=0}^{i} q'_j. \tag{4}$$

The I^{2}NOE problem under IC model can be formulated as follows:

$$\max \sigma(A) \tag{5}$$

subject to:

$$\sum_{i=1}^{m} w_i x_i \le B, \tag{6}$$

$$q'_i = \left(1 + \frac{x_i}{|N^{\mathrm{in}}(v_j)|}\right) q_i, \tag{7}$$

$$q'_i \le 1, \tag{8}$$

$$x_i \in \{0, 1, \ldots\}. \tag{9}$$

In the above formulated problem statement, each node $v_i \in V$ has a corresponding variable x_i which represents how many times v_i should be selected. Constraint (5) ensures that the total investment is not greater than budget B, constraint (6) represents the relationship between the original and final influence from v_i to v_j.

3.2. $\mathcal{NP}$-hardness of I^{2}NOE problem

Before trying to solve the proposed I^{2}NOE problem, we analyze the complexity of the problem in this subsection.

Theorem 1. *I^{2}NOE problem is $\mathcal{NP}$-hard.*

Proof. To prove this, we reduce I^{2}NOE problem from the bounded knapsack problem (BKP), which is a well-known $\mathcal{NP}$-complete problem in combinatorial optimization: Given a set with n items, each item i has a weight w_i and a value c_i, determine a collection, in which the number of each item is less than or equal to b_i, such that the collection's total weight is not greater than a given limit W and its total value is as large as possible. A BKP can be formulated as follows:

$$\max \sum_{i=1}^{n} c_i x_i \tag{10}$$

subject to:

$$\sum_{i=1}^{n} w_i x_i \le W, \tag{11}$$

$$x_i \in \{0, 1, \ldots, b_i\}. \tag{12}$$

For a graph $G = (V, E)$, a subset D of V is a dominating set of G if each node not in D is adjacent to some node in D. We consider the I^{2}NOE problem under a certain case where the seed set A is a dominating set of TC. Under this case, for every node v satisfying $v \in V$ and $v \notin A, h(A, v) = 1$, which means every $P \in \mathcal{P}(A, v)$ only contains one hop. Based on this, $q'_j = (1 + \frac{x_j}{|N^{\text{in}}(v_j)|})q_j$, thus the objective function of (5) equals to $\sum_{G' \in \Gamma_G} \text{Prob}(G') \sum_{i=0}^{n-1} a_{G'}(A, i) \prod_{j=0}^{i} q_j + \sum_{G' \in \Gamma_G} \text{Prob}(G') \sum_{i=0}^{n-1} a_{G'}(A, i) \prod_{j=0}^{i} \frac{q_j}{|N^{\text{in}}(v_j)|} x_j$. Since q_j and $|N^{\text{in}}(v_j)|$ are constants, (5) only contains one degree terms of x_j which can be expressed as $\sum c_i x_i$ and constants, thus (5) is the same as (10). Furthermore, each w_i in I^{2}NOE problem can be regarded as same as each w_i in BKP, (6) in I^{2}NOE problem and (11) in BKP are the same.

From (7) and (8), we obtain that $x_i \le \lfloor \frac{1-q(u)}{q(u)} |N^{\text{in}}(u)| \rfloor$, which shows the upper bound of x_i. Thus, when the seed set A is a dominating set of the social network, the I^{2}NOE problem is equivalent to a BKP. Since BKP is $\mathcal{NP}$-complete, I^{2}NOE problem is $\mathcal{NP}$-hard. □

4. Methodology

In this section, we will propose the algorithm *ABICN* for I^{2}NOE problem. In *ABICN*, to estimate the $\mathcal{P}(A, v)$ mentioned in Table 1, we will design the *CIICN* algorithm. The analysis of time complexity will also be presented.

4.1. *Algorithms for I^{2}NOE problem*

We propose *CIICN* algorithm to estimate the influence, i.e., $\mathcal{P}(A, v)$ step by step according to IC-N model's property. On the other hand, since we have proved that the I^{2}NOE problem is $\mathcal{NP}$-hard, we devise a greedy approximation algorithm named *ABICN*, which allocates the individual u that has the maximum positive opinion value of $\frac{\text{inf}(u)|N^{\text{out}}(u)|}{w(u)}$ with k times. In choosing the positive nodes with maximum $\frac{\text{inf}(u)|N^{\text{out}}(u)|}{w(u)}$, if more than one node shares the same maximum value, we compare the influence of their out-neighbors, and choose the one with the largest influence.

Algorithm 1 *CIICN*: Algorithm for computing influence

1: **INPUT**: $G = (V, E, p, q)$, A, two sets U and U'.
2: **OUTPUT**: $\inf(v_i)$ and $\inf(G)$.
3: Let $U = A$; $U' = V - A$;
4: **while** U' is not empty **do**
5:   **for each** $u \in U$ that has positive attitude
6:     **for each** $v_j \in N^{\text{out}}(u) \cap U'$
7:       $\inf(v_j) = \inf(v_i)(1-$
$(\prod_{P_k \in P(A,v_j)}(1 - \prod_{(v_i,v_j)\in P_k} p(v_i, v_j))) \prod_{j=0}^{i} q_j$;
8:     **end for**
9:   **end for**
10:   $U = U \cup \{v_j\}$;
11:   $U' = U' - \{v_j\}$;
12: **end while**
13: **for each** $v_i \in U$
14: $\inf(G) = \inf(G) + \inf(v_i)$;
15: **end for**
16: **return** $\inf(v_i)$ and $\inf(G)$.

According to constraints (7) and (8), it is easy to find that k is x_i, and we have $q(u)(1+\frac{k}{|N^{\text{in}}(u)|}) \leq 1$, which implies $k \leq \lfloor \frac{1-q(u)}{q(u)} |N^{\text{in}}(u)| \rfloor$. The details of the solution for I^2NOE problem are described in Algorithms 1 and 2.

4.2. *Algorithm complexity*

Let n be the size of the social network. Note that in *CIICN* algorithm we have to obtain the shortest path between any pair of nodes in order to update $\inf(v_i)$, and it will cost $O(n^3)$ running time. From steps 4–6 in *CIICN*, we use nested loops with three layers, and as a whole these loops take $O(n^3)$ running time. Hence, *CIICN* has time complexity of $O(n^3) + O(n^3)$, which is still $O(n^3)$.

ABICN is a greedy approximation algorithm for the I^2NOE problem. *ABICN* has nested loops with two layers (the running time of the loop from steps 8–10 is short and can be ignored), each time the inner loop iterates it takes $O(n^3)$ since *CIICN* algorithm is called once, and the outer loop iterates for n times at most. Therefore, the time complexity of *ABICN* is $O(n \cdot n^3) = O(n^4)$.

5. Experiments

We have done experiments on three real-world data sets: WikiVote, NetHEPT and Enron Email, and compared our algorithms with three algorithms: random method, degree centrality and weight centrality based on the three data sets. In this section we will present the experimental results and the analysis.

Algorithm 2 *ABICN*: Algorithm for allocating budget

INPUT: $\inf(v_i)$, $w(v_i)$, budget B, a set H, k and g.
OUTPUT: The budget $\widetilde{w}(u)$ allocated to u.
Let $H = V - A$; $k = 0$; $g = 0.0$;
while $B > 0$ and H is not empty do
  for each $v_i \in H$
    select the node u with positive opinion and has the maximum value of $\frac{\inf(v_i)|N^{\text{out}}(u)|}{w(v_i)}$;
    if there are multiple nodes u
      for each $u' \in N^{\text{out}}(u)$
        $g = g + \inf(u) * p(u, u')$;
      end for
      select the node u with the maximum value g;
    end if
  end for
  $k = \lfloor \frac{1-q(u)}{q(u)} |N^{\text{in}}(u)| \rfloor$;
  $\widetilde{w}(u) = k \cdot w(u)$;
  allocate u with $\widetilde{w}(u)$;
  $H = H - \{u\}$;
  $B = B - k \cdot w(u)$;
  for each $v_j \in N^{\text{out}}(u)$
    $q(u) = q(u)(1 + \frac{k}{|N^{\text{in}}(u)|})$;
  end for
  update the influence using the *CIICN* algorithm;
end while
return $\widetilde{w}(u)$.

Table 2. Data Description.

Data sets	WikiVote	NetHEPT	Enron Email
No. of nodes	7K	15K	36K
No. of edges	101K	31K	367K
Avg. degree	26.6	4.1	10
No. of communities	24	1820	1256
No. of Largest community size	7066	1251	5053
Avg. community size	296.5	8.4	29.2

5.1. *Data settings*

In the three realistic data sets: WikiVote, NetHEPT and Enron Email, whose statistics are plotted in Table 2, we set a probability from 0.01 to 0.1 randomly, and uniformly assign the node weight from the set $\{10, 20, \ldots, 80\}$ at random, we also set $q(v)$ from 0.6 to 0.8 randomly. In the experiments, we select the nodes with the largest degrees as the seeds. The data of the experiments is described as follows:

WikiVote. Taken from the inception of Wikipedia till January 2008. Nodes in the network represent wikipedia users and a directed edge from node u to node v represents that user u voted on user v. The data set is publicly available at http://snap.stanford.edu/data/wiki-Vote.html.

NetHEPT. Its data is from the "High Energy Physics (Theory)" section (from 1991 to 2003) of arXiv (http://arXiv.org). The nodes and edges in NetHEPT denote the authors and the co-authorship respectively.

Enron Email. Extracted from Enron corpus, which is a large scale email collection from a real organization over a period of 3.5 years. The nodes in the network are email addresses, and an edge from node i to node j means user i sends at least one email to user j. (http://snap.stanford.edu/data/email-Enron.html).

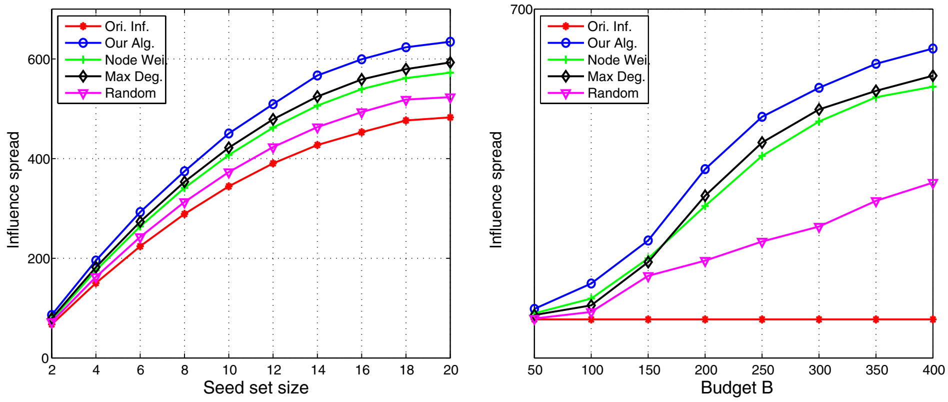
5.2. *Algorithms for comparison*

In this part, we describe three comparison algorithms: random, degree centrality and weight centrality. Random algorithm is a basic comparison method, in order to measure the performance of other methods. Max Degree method could improve more influence since the larger degree node has more neighbors, and Node Weight centrality could choose more nodes to allocate the budget in order to improve more influence. Moreover, we denote the influence before the budget B is allocated to enhance it as Original Influence.

(1) *Random*: In the problem, we choose the nodes v randomly when the budget B has not been allocated, as long as satisfy $q(v) \leq 1$ after the improvement.
(2) *Max Degree method* (Max Deg.): This is a greedy method. The strategy prefers the positive opinion node with larger degree, which means selecting such nodes will improve the influence of many neighbor nodes. We select the inactivated nodes with maximum degree when the budget B has not been allocated.
(3) *Node Weight centrality* (Node Wei.): This is a simple heuristic algorithm, we prefer to choose the positive opinion nodes with small node weight, which means we can choose more nodes to allocate the budget.
(4) *Original Influence* (Ori. Inf.): This is the positive influence before the budget B is allocated.
(5) *Our Algorithms* (Our Alg.).

5.3. *Experimental results*

In this section we analyze the performance of our algorithm. To compare with other algorithms, we concern about the influence spread when the seed set size and budget B gradually increase separatively. We compute the positive influence, and conduct our algorithms and other three comparison methods. For each comparison, we run each algorithm 1000 times to get the average value. For ease of reading, the legend

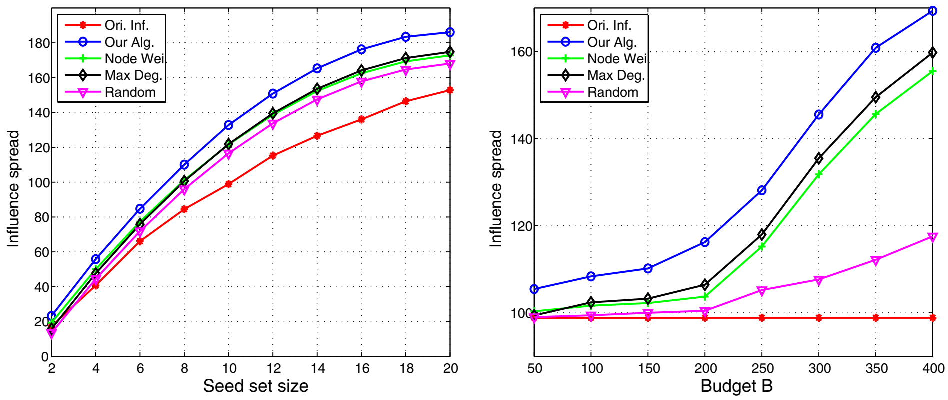


(a) Influence spread (b) Budget allocate

Fig. 1. WikiVote on IC-N model.

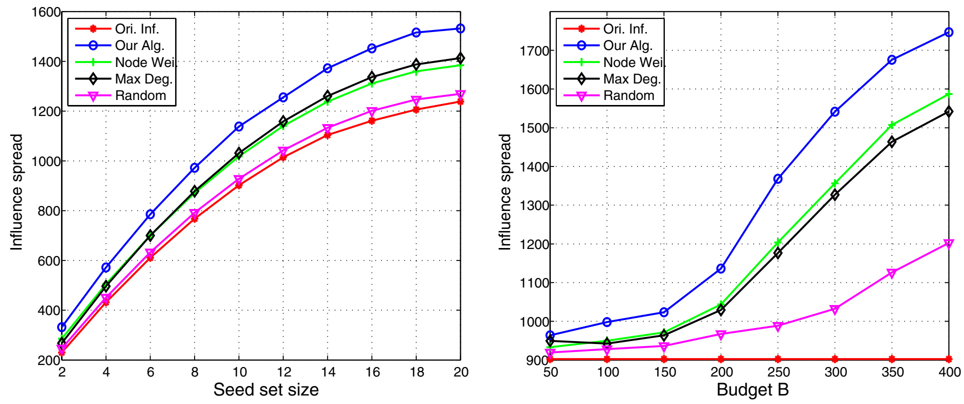
of each figure lists the algorithms in the same order as the influence spread with 20 seeds and budget as 400.

From Fig. 1(a), it is obvious that the original influence spread increases as the seed set size increases from 2 to 20, and the influence spread of our algorithm increases fast. However, the increasing speed becomes slower with the increment of the number of seeds. In Fig. 1(b), we allocate 10 seeds into the seed set, then the influence spread increases rapidly with the budget B increases. When the budget reaches 250, the increase speed becomes slow. Our algorithm is superior to the other three methods.



(a) Influence spread (b) Budget allocate

Fig. 2. NetHEPT on IC-N model.



(a) Influence spread

(b) Budget allocate

Fig. 3. Enron Email on IC-N model.

In Fig. 2, we conduct the algorithms on NetHEPT under IC-N model, and set the budget and seed set size as 200 and 10, respectively. The growth trend of influence spread under IC-N model is similar to Fig. 1.

Similarly, we run the algorithms on Enron Email. Through Figs. 1–3, the results exhibit that as the seed number and budget further increase, the influence spread increases, and our algorithm is generally better than other three methods.

6. Conclusion

In this paper, we address the problem of improving influence when negative opinions emerge in social networks. Our strategy is to increase the investment of members under bounded budget so that the influence can be improved. We study the problem under IC-N model, and prove that this problem is $\mathcal{NP}$-hard. After that we propose *CIICN* and *ABICN* algorithms for IC-N model. To test the efficiency of our algorithms, we have conducted extensive experiments over three large-scale social networks in real world, and compared our algorithm with three algorithms including random, degree links and weight links. The results show that our algorithms outperform these three algorithms.

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