Construction of Higher Spectral Efficiency Virtual Backbone in Wireless Networks

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Abstract

One promising solution to improve the efficiency of wireless networks is to control the number of nodes involved in multi-hop routing by employing virtual backbone. On the one hand, a virtual backbone becomes more efficient as its size is getting smaller. However, as the size of a virtual backbone is getting smaller, the throughput of the virtual backbone is degraded since the length of a routing path between a pair of nodes through the virtual backbone can be much longer than their hop distance in the original network. Due to the reason, several efforts are recently made to identify a virtual backbone including a shortest path between every pair of nodes in the original graph. In this paper, we investigate a new strategy to compute higher throughput virtual backbone in wireless networks. We employ a new information theoretic metric called *spectral-efficiency* by Chen *et al.* and propose a new virtual backbone computation algorithm in homogeneous wireless networks with some interesting theoretical analysis. Our simulation results indicate our algorithm produces a virtual backbone with higher spectral-efficiency than the existing alternatives. We also conduct another simulation using OMNet++ and show the virtual backbone produced by our algorithm has the highest throughput.

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Keywords:
wireless networks, virtual backbone, shortest path, higher throughput, spectral-efficiency.

1. Introduction

Recently, the concept of virtual backbone has been introduced to improve the efficiency and performance of wireless networks. The main idea of this approach is to establish a connected subset of nodes in a wireless network such that any two nodes can communicate with each other through the nodes in the subset. In essence, this approach decreases the number of nodes involved in message routing and reduces the amount of signal collision and interference. Meanwhile, it is apparent that such benefits can be magnified as the size of the subset becomes smaller. In [5], Guha and Khuller formulated the problem of computing a smallest virtual backbone in a given network graph as the minimum *connected dominating set (CDS)* problem. Since the minimum CDS problem is NP-hard in various network graphs, many efforts are made to introduce approximation algorithms for the problem.

Over years, the network community has tried to introduce a new metric for better routing algorithms. Frequently, these algorithms are built on a link-layer level abstraction of network, which does not fully consider the impact of the physical layer. Therefore, those algorithms do not concern about the fundamental performance limits of wireless communication such as spectral-efficiency. However, spectral-efficiency is an important concept in wireless networks since higher spectral-efficiency means higher network throughput [1, 2]. In [31], the authors showed that given an one-dimensional linear network, there exists an optimal number of hops, which is not necessarily the shortest hop, in terms of maximizing end-to-end spectral-efficiency. In [28], Chen *et al.* introduced a new metric to find a spectral-efficient routing path in multi-hop wireless networks and proposed spectral-efficient routing algorithms.

To the best of our knowledge, there have been five research papers concerning about the routing cost in the construction of virtual backbone [17, 16, 19, 20], and all of them share the same motivation: the cost of routing over a virtual backbone computed by a minimum CDS algorithm can be high. This is because any two nodes which are very near to each other in the original network graph may need to communication with each other using a very long routing path through the virtual backbone, which will degrade the performance of the wireless network.

In this paper, we investigate how to improve the throughput of a virtual backbone in wireless networks. We observe that all of the existing CDS computation algorithms with routing cost consideration use the number of hops as the cost metric of a routing path. Inspired by Chen *et al.* work which showed a path with higher spectral-efficiency is with higher throughput than a shortest hop path, we propose a new virtual backbone computation algorithm incorporating their spectral-efficiency metric so that we can obtain a virtual backbone with higher throughput. The contributions of this paper are as follows. First, based on the spectral-efficiency metric in [28], we define the *maximum spectral-efficient connected dominating set (MSE-CDS)* problem whose goal is to compute the most spectral-efficient virtual backbone in homogenous wireless networks. We also show this problem is NP-hard. Second, we propose a new algorithm for MSE-CDS, namely *spectrum-efficient virtual backbone generater (SE-VBG)*. We show the correctness of our algorithm and provide the proofs of some interesting characteristics of the algorithm. Third, via simulation, we compare the average performance of our algorithm against the existing competitors. Our simulation results show SE-VBG produces a CDS with higher spectral-efficiency. Last, using a comprehensive simulator, OMNet++, we show the virtual backbone produced by our algorithm is with higher throughput than the existing competitors. As a result, *our algorithm outperforms the competitors in terms of throughput, a performance metric of real importance, instead of the other existing abstract performance metrics, the size of virtual backbone and the average hop distance.*

The rest of the paper is organized as follows. Related work is given in Section 2. In Section 3, we provide some preliminaries. The formal definition of MSE-CDS is given in Section 4. The description of our algorithm SE-VBG and corresponding analysis are in Section 5. We present the simulation results and corresponding discussions in Section 6 and conclude this paper in Section 7.

2. Related Works

Over many years, virtual backbone has been studied as a promising approach to improve the efficiency of wireless networks. Since the benefit of the virtual backbone can be augmented as its size becomes smaller, many efforts are made to find smaller size virtual backbone. It is well-known that the minimum CDS problem is NP-hard in unit disk graph (UDG), and thus many people focused on the design of approximation algorithm for this problem [5, 14, 15, 7]. In most cases, a *dominating set (DS)* is identified and some more additional nodes whose number is bounded by a constant factor of the DS are added to make the nodes in the DS connected. Frequently, a *maximal independent set (MIS)* is used as an approximation of the minimum DS problem, which is also NP-hard. In [8], Wan *et al.* showed the the size of an MIS is bound by $4 \cdot |opt_{MCDS}| + 1$, where $|opt_{MCDS}|$ is the size of an optimal CDS opt_{MCDS}, for the first time. Later, the bound is tightened by a series of attempts such as [9, 10, 11, 12, 13]. To the best of our knowledge, the most tight bound is in [12], which states the size of any MIS is bounded by $3.4306 \cdot |opt_{MCDS}| + 4.8185$.

One crucial performance issue in the virtual backbone based routing is that the virtual backbone may not include the shortest path between a pair of nodes. As a result, any two nodes which are only a few hops far in the original network may need to communicate through a number of intermediate virtual backbone nodes. Clearly, this can reduce the performance (i.e. throughput) of the wireless networks adopting virtual backbone. In [18], Kim *et al.* discussed the importance of routing cost in virtual backbone construction for the first time. They also studied a joint optimization problem of minimizing the size of CDS and the diameter of CDS, and proposed a centralized algorithm and a distributed algorithm which have a constant factor approximation ratio for each of the optimization goals. In [17], Ding *et al.* introduced a polynomial time exact algorithm to compute a minimum size CDS including every pair of shortest paths between each pair of nodes in a given general graph. In [16], Ding *et al.* proposed a $(1 - \ln 2) + 2 \ln \delta$-approximation algorithm for the minimum routing cost CDS problem, whose goal is to find a minimum size CDS of a given general graph including at least one shortest path between every pair of nodes in the graph. Later, Ding *et al.* also extended this result into a wireless networks with directional antennas [19]. In very recent report by Du *et al.*, the problem of computing a minimum CDS including a path for each pair of nodes whose length is bound by a constant factor α of the shortest path length between the nodes in the original UDG is studied [20], in which the authors proposed a centralized algorithm and a distributed algorithm for the problem and prove that the size of an output of their algorithms is bounded by $148 \cdot |opt_{MCDS}| + 208$ and α is in fact 7 in their algorithms.

Over many years, several metrics for routing algorithms in wireless networks have been proposed by both information theory community [22][34] and networking community [25][26][27]. In [28], Chen *et al.* pointed out the results from information theory community is too complicated to use in practice and the results from networking community mostly focus on hop distance, which does not fully consider the impact of the physical layer. Previously, the authors in [31][23] found that there is a path with an optimal number of hops, not necessarily the shortest path, in terms of maximizing end-to-end spectral-efficiency. Based on this result, Chen *et al.* considered the problem of computing a maximum spectrum-efficiency routing path in multi-hop wireless networks, with the constraint of equal bandwidth sharing, and proposed two efficient heuristics for sub-optimal solution. In [29], Sadd improved this result and proposed a polynomial time optimal algorithm for this problem. In this paper, we investigate if Chen *et al.*'s result can be applied to the design of virtual backbone to improve its spectral-efficiency and consequently its throughput.

3. Preliminaries

In this section, we briefly introduce the network model, channel model, and spectrum-efficient routing metric. In [28], the authors claim that from the information theoretical point of view, any two nodes can communicate with each other with a sufficiently low rate and thus they use a connected graph to abstract a given wireless network, and studied the spectral-efficient routing problem over the graph. However, in practice, if two nodes are far enough, the communication link between them is too inefficient to be used in practice. Therefore, we assume that any two nodes whose distance is greater than a threshold level is disconnected and use UDG $G = (V, E)$ to abstract a wireless network, where V is the set of nodes and E is the set of edges between every pair of nodes. During the rest of this paper, $N = |V|$ and $M = |E|$. For each communication link $e \in E$, $s(e)$ and $r(e)$ are the sender end and receiver end of e, respectively. Also, $|e|$ is the Euclidean distance of the link e. A path L from a node v_0 to another node v_n consists of a sequence of distinct links $e_1, e_2, \cdots, e_m \in E$ such that (i) $s(e_1) = v_0$, (ii) $r(e_m) = v_n$, and (iii) $r(e_{j-1}) = s(e_j)$, where $2 \leq j \leq m$. $|L|$ is the length of L, which is the number m of hops in the path.

This paper adopts a standard path-loss model such that the path-loss factor over an edge e is given by $G_e = c[\max(|e|, D_f)]^{-\alpha}$, where D_f is the far-field distance [35], α is the path-loss exponent (typically between 2 to 4), and c is a constant. Since $|e|$ is much greater than D_f in reality, we can simplify the path-loss model as $G_e \approx c|e|^{-\alpha}$. We properly normalize the transmission power of the nodes and $c = 1$, which are good enough for relative performance comparison. In addition, we also assume additive white Gaussian noise N_0, which corrupts the signal received by each receiving end of the links in the network, is equivalent for all receivers. We further follow the system model of Haenggi and Puccinelli [36], and assume that the transmission power of all senders is equally P. At last, we assume that the wireless communication links in the network has a finite bandwidth B. Under the assumptions made so far, the network *signal-to-noise ratio (SNR)* can be defined as $SNR_e = \frac{PG_e}{N_0 B} = \frac{P}{N_0 B|e|^\alpha}$. By [23], given optimum bandwidth allocation among links, the maximum spectral efficiency along L is

$$\Big(\sum_{e \in L} \frac{1}{\log{(1 + SNR_e)}}\Big)^{-1},$$

which will be maximized when $\sum_{e \in L} \frac{1}{\log{(1+SNR_e)}}$ becomes minimized since $\log{(1 + SNR_e)} > 0$. Then, from [37], under the restriction of equal bandwidth sharing, the end-to-end spectral efficiency of L is

$$SE(L) = \min_{e \in L} \Big((1/|L|) \log{(1 + SNR_e)}\Big), \tag{1}$$

where $1/|L|$ is multiplied due to the sharing of bandwidth among relay links. The implication of Eq. (1) is that for a path L, the quality of signal corresponds to the worst SNR of a link in L ($SNR_{e^*} = \min_{e \in L} SNR_e$) and the bandwidth use is proportional to the inverse of the number of hop, $|L|$. The value of the spectral efficiency Eq. (1) increases as SNR_{e^*} increases or $|L|$ decreases. For the routing paths connecting a given source and destination, if $|L|$ increases (or decreases), there are more (or less) relay nodes and SNR_{e^*} is more likely to increase (or decrease) due to shorter (longer) inter-relay distances. During the rest of this paper, we define the weight of a path L as

$$weight(L) = \min_{e \in L} \log{(1 + SNR_e)}, \tag{2}$$

and using this, we can simply Eq. (1) to

$$SE(L) = weight(L)/|L|. \tag{3}$$

4. Maximum Spectral-Efficient Connected Dominating Set (MSE-CDS) Problem

In the literature, it is widely believed that the smaller the size of a virtual backbone is, the better the performance of a wireless network employing the virtual backbone can be boosted since the backbone will incur less overhead for routing and thus the network will suffer less from wireless signal collision and interference. Recently, several efforts are made to incorporate the cost of routing as another metric to form a virtual backbone in wireless networks. The motivation of the series of researches is that when the size of a virtual backbone is minimized, the length of routing path between a pair of nodes over the virtual backbone can be much longer than the length of shortest path between them in the original graph.

In this paper, we focus on the computation of CDS to form a virtual backbone in homogeneous wireless network (e.g. a wireless network of a number of nodes with the same physical performance) with the goal of achieving higher throughput. Motivated by Chen *et al.*'s work, we employ spectral-efficiency as a metric instead of hop distance and strive to design a CDS computation algorithm generating a virtual backbone with higher throughput.

Now, we present some notations and provide the formal definition of our problem of interest, MSE-CDS. Given two nodes $v_i, v_j \in V$, P_{ij} is the set of paths between the two nodes in G. Also, $p^*_{ij} \in P_{ij}$ is the maximum spectral-efficient path between v_i and v_j. Then, from Eq. (3), we can define

$$SE(p^*_{ij}) = \max_{p_{ij} \in P_{ij}} SE(p_{ij}). \tag{4}$$

Definition 1 (MSE-CDS)**.** *Given a wireless network abstracted as a connected UDG $G = (V, E)$, the maximum spectral-efficient connected dominating set (MSE-CDS) problem is to find a minimum cardinality node set $D \subseteq V$ such that (i) $\forall u \in V \backslash D$, $\exists v \in D$ such that $(u, v) \in E$, (ii) the induced graph $G[D]$ is connected, and (iii) $\forall v_i, v_j \in V$, $p^*_{ij} \backslash \{v_i, v_j\} \subseteq D$, where p^*_{ij} is a maximum spectral-efficiency path between v_i and v_j in G.*

Definition 1 dictates that the goal of MSE-CDS is to compute a minimum CDS D of G including the most spectral-efficient path between every pair of nodes in G. Now, we show that MSE-CDS is NP-hard.

Theorem 1. *The MSE-CDS problem is NP-hard.*

Proof. We prove MSE-CDS is NP-hard by showing that in $\tilde{G}$, a subclass of UDG in which the lengths of all the edges are the same, MSE-CDS is equivalent to a known NP-hard problem, the *minimum routing cost connected dominating set (MOC-CDS)* problem in [16]. First of all, observe that in $\tilde{G}$, all paths between any pair of nodes have the same path weight which is defined in Eq. (2). Therefore, for any pair of nodes in $\tilde{G}$, the maximum spectral-efficiency routing path between them (defined in Eq. (3)) is equivalent to the shortest path. Since the goal of MOC-CDS is to find a minimum cardinality CDS which includes at least one shortest path for every pair of nodes in a given connected UDG, MSE-CDS is equivalent to MOC-CDS in $\tilde{G}$. As a result, this theorem holds true. □

5. SE-VBG: A New Heuristic for MSE-CDS

Let us first introduce a new centralized algorithm for MSE-CDS, namely *spectrum-efficient virtual backbone generater (SE-VBG)*, and provide the analysis of some interesting characteristics of the algorithm. Consider a connected UDG $G = (V, E)$ which is an abstraction of a given homogenous wireless network. For each node $v_i \in V$, we define the set N_i of neighbors of v_i in G, i.e. $N_i = \{v_j | v_j \in V \text{ and } (v_i, v_j) \in E\}$. Also, for each pair of nodes v_i and v_j, we define a shortest path between the nodes as $sp(v_i, v_j)$ and its hop count as $|sp(v_i, v_j)|$. Initially, G has a weight neither on edge nor node. However, from now on, we assume that each edge $e \in E$ has a weight of $WL(e) = \log(1 + SNR_e)$, and each node $v_i \in V$ has a weight of $WN(v_i) = \min_{v_j \in N_i} WL(v_i, v_j)$, where $WL(v_i, v_j)$ is the weight of the edge between v_i and v_j. At last, we use Eq. (2) as the weight of a path L, i.e. $weight(L) = \min_{e \in L} \log(1 + SNR_e)$. Now, we introduce two key lemmas, Lemma 1 and Lemma 2, in the design of our algorithm for MSE-CDS.

Lemma 1 ([29])**.** *Consider a connected UDG $G = (V, E)$. For any pair of nodes $v_i, v_j \in V$, the problem of computing a maximal spectral efficiency route p_{ij}^* from v_i to v_j in G satisfying Eq. (4) is equivalent to computing a path $p_{ij}^{w^*}$ such that*

$$SE(p_{ij}^{w^*}) = \max_{w \in WL} \Big[\max_{p_{ij} \in P_{ij}, weight(p_{ij}) \geq w} SE(p_{ij}) \Big], \tag{5}$$

where $WL = \{WL(e) | \forall e \in E\}$.

Lemma 2. *The problem of computing $p_{ij}^{w^*}$ in Lemma 1 is equivalent to the problem of computing p_{ij}^{sp} such that*

$$SE(p_{ij}^{sp}) = \max_{w \in WL_{sp}} \Big[\max_{p_{ij} \in P_{ij}, weight(p_{ij}) \geq w} SE(p_{ij}) \Big], \tag{6}$$

where $WL_{sp} = \{WL(e) | WL(e) \geq weight(sp(v_i, v_j)), e \in E\}$.

Algorithm 1 SE-VBG ($G = (V, E)$)

Set $I \leftarrow \emptyset$, $C \leftarrow \emptyset$, and $D \leftarrow \emptyset$. For each link $e \in E$, compute $WL(e) = \log(1 + SNR_e)$.

/* **Step 1 Begins** */

Compute an MIS I of G. For each node $b_i \in I$, set $I_i = \{v | v \in V \backslash I$ and $(b_i, v) \in E\}$.

/* **Step 2 Begins** */

For each pair of $b_i, b_j \in I$ $(i \neq j)$, find the shortest path $sp(b_i, b_j)$ between them and calculate the weight of the shortest path based on Eq. (2).

For each node $b_i \in I$, compute its weight $WN(b_i) = \min_{v \in I_i}\{WL(b_i, v)\}$.

For each pair of $b_i, b_j \in I$ $(i \neq j)$, $weight(b_i, b_j)^* = \min\{weight(b_i, b_j)^{sp}, WN(b_i), WN(b_j)\}$.

/* **Step 3 Begins** */

Set $E_{temp} \leftarrow \emptyset$, and for each pair of $b_i, b_j \in I; i \neq j$, $P_{ij}^{temp} \leftarrow \emptyset$.

for each pair of $b_i, b_j \in I; i \neq j$ **do**

  for each link $e \in E$ **do**

    if $WL(e) = weight(b_i, b_j)^*$ **then**

      $P_{ij}^{temp} \leftarrow P_{ij}^{temp} \bigcup \{sp(b_i, b_j)\}$ and record $SE(sp(b_i, b_j)) = \frac{weight(b_i,b_j)^{sp}}{|sp(b_i,b_j)|}$.

    else if $WL(e) > weight(b_i, b_j)^*$ **then**

      $E_{temp} \leftarrow E \backslash \{e | e \in E$ and $WL(e) < weight(b_i, b_j)^*\}$

      Find the shortest path $sp'(b_i, b_j)$ between b_i and b_j in $G[E_{temp}]$, and $P_{ij}^{temp} \leftarrow P_{ij}^{temp} \bigcup \{sp'(b_i, b_j)\}$ and record $SE(sp'(b_i, b_j)) = \frac{weight(b_i,b_j)^{sp'}}{|sp'(b_i,b_j)|}$.

    end if/* we do not consider the case $WL(e) < weight(b_i, b_j)^*$ */

  end for

  Choose the path $\tilde{p}(b_i, b_j)$ with the largest value of SE in P_{ij}^{temp}, and $C \leftarrow C \bigcup (\tilde{p}(b_i, b_j) \backslash \{b_i, b_j\})$.

end for

Return $D = I \bigcup C$.

Proof. We first rewrite Eq. (5) to

$$SE(p_{ij}^{w^*}) = \max_{w \in WL} SE^w(p_{ij}), \text{where}$$

$$SE^w(p_{ij}) = \max_{p_{ij} \in P_{ij}, weight(p_{ij}) \geq w} SE(p_{ij}),$$

and $WL = \{WL(e)|\forall e \in E\}$. Similarly, Eq. (6) can be rewritten as

$$SE(p_{ij}^{sp}) = \max_{w \in WL_{sp}} SE^w(p_{ij}) \text{ and } WL_{sp}$$

$= \{WL(e)|WL(e) \geq weight(sp(v_i, v_j)), e \in E\}$. Now, consider $p_{ij}^{w^*}$. Then, by Eq. (5), there should be some $w \in WL$ such that $w \leq weight(p_{ij}^*)$. In addition, by the definition of spectral-efficiency, we have $SE(sp(v_i, v_j)) \leq SE(p_{ij}^*)$, which implies

$$\frac{weight(sp(v_i, v_j))}{|sp(v_i, v_j)|} \leq \frac{weight(p_{ij}^*)}{|p_{ij}^*|}.$$

Since $sp(v_i, v_j)$ is the shortest path between v_i and v_j, we also have $|sp(v_i, v_j)| \leq |p_{ij}^*|$, which implies

$$\min_{w \in WL} w \leq weight(sp(v_i, v_j)) \leq weight(p_{ij}^*).$$

Furthermore, the spectral-efficiency of a path is proportional to the weight of the path. Therefore,

$$\begin{aligned} &SE(p_{ij}^{w^*}) \\ &\quad = \max\Big[\max_{w \in WL_{sp}} SE^w(p_{ij}), \max_{w \in WL \setminus WL_{sp}} SE^w(p_{ij})\Big] \\ &\quad = \max_{w \in WL_{sp}} SE^w(p_{ij}) = SE(p_{ij}^{sp}), \end{aligned} \tag{7}$$

which completes the proof. □

Based on the lemmas, we now introduce the *spectrum-efficient virtual backbone generater (SE-VBG)* algorithm, which is a polynomial time centralized approximation for MSE-CDS. Algorithm 1 is the detail of SE-VBG. In the preliminary phase (Line 1 in Algorithm 1), we compute the weight of each edge, which can be finished in $O(|E|)$ time. Largely, SE-VBG consists of the following three steps.

Step 1. We compute an MIS I of a given UDG G via the classical scheme in [8] with the time complexity of $O(|V|)$. For each node $b_i \in I$, we also construct the set $I_i \subseteq V \setminus I$ of the neighbors of b_i in G. It is easy to find that this step spends $O(|I|)$ time.

Step 2. For each pair of nodes in I, we establish a *referential (tentative) path weight*, which will be used as a guideline later to find a spectrum-efficient routing path for each pair of nodes in I so that the union of I and the nodes in the path for each pair of nodes in I can form a CDS. That is, for each pair $b_i, b_j \in I$, we select

$$weight(b_i, b_j)^* = \min\left[weight(b_i, b_j)^{sp}, WN(b_i), WN(b_j)\right]$$

as the minimum achievable spectral-efficiency of a path between b_i and b_j. This process has the time complexity of $O(|I|^2)$. $WN(b_i)$ and $WN(b_j)$ are being considered since the spectral efficiency of a route path in CDS with b_i and b_j as two end points is at least the minimum of them. We also consider $weight(b_i, b_j)^{sp}$ since the maximum spectral efficiency of a path connecting b_i and b_j can be smaller than $WN(b_i)$ and $WN(b_j)$.

Since the end-to-end spectral efficiency of a path is inversely proportional to the length of the path (Eq. (3)) and we cannot find all paths between every b_i and b_j within polynomial time. Therefore, we rather compute the shortest path between each pair of nodes as well as consider corresponding path weight to obtain a possible referential path weight. Remind that the weight $weight(b_i, b_j)^{sp}$ of a shortest path between two nodes $b_i, b_j \in I$ is the minimum weight of a link in the path.

Step 3. In this step, we find a subset C of nodes in $V \setminus I$ as the connecters for I such that $I \cup C$ is connected. Our strategy for this step is that for each pair of nodes $b_i, b_j \in I$, we identify a spectral-efficient path and add it to C. By repeating this procedure for each pair of nodes, an empty subset C will includes sufficient nodes such that $I \cup C$ is a CDS. For each pair $b_i, b_j \in I$, we pick a set of links in E based on $weight(b_i, b_j)^*$ such that the union of the links can connect b_i and b_j as follows. First, we compare the weight of each link in E, $WL(e)$, with $weight(b_i, b_j)^*$. Then, there are two possible cases.

Case 1. $WL(e) = weight(b_i, b_j)^*$. In this case, $WL(e)$ is equivalent to the minimum of $weight(b_i, b_j)^{sp}$, $WN(b_i)$, and $WN(b_j)$. Since $sp(b_i, b_j)$ is with the minimal path length and the end-to-end spectral efficiency of a path is inversely proportional to the path length, $sp(b_i, b_j)$ has the maximum spectral efficiency among all the paths between b_i and b_j. Therefore, we choose $sp(b_i, b_j)$ to be a candidate path between b_i and b_j.

Case 2. $WL(e) > weight(b_i, b_j)^*$. In this case, we consider $WL(e)$ as the lower bound of link weight on the candidate path that we are looking for, and compute the shortest path $sp'(b_i, b_j)$ within $G[E_{temp}]$, where $E_{temp} = E \setminus \{e | e \in E \text{ and } WL(e) < weight(b_i, b_j)^*\}$. Since the new shortest path in $G[E_{temp}]$ is with the minimal path length, this path is the one with the maximum spectrum-efficiency in the subgraph.

Note that the operation in Step 3 is virtually an iterative use of a shortest path procedure. Furthermore, the case $WL(e) < weight(b_i, b_j)^*$ is out of consideration here based on Lemma 2. Finally, we choose the path with largest spectral efficiency among the candidate paths for b_i and b_j and add the intermediate nodes on the selected path to C. We can find that this step has the time complexity of $O(|I|^2|E|)$.

On the whole, the time complexity of Algorithm 1 is $O(|V|^4)$. Now, we provide the proof of correctness of Algorithm 1 for MSE-CDS as well as the theoretical analysis of two interesting properties of the algorithm.

Theorem 2. *Given a connected UDG $G = (V, E)$, the output D of Algorithm 1 is a CDS satisfying three rules in Definition 1.*

Proof. We first show the node set D obtained by Algorithm 1 meets the first two conditions in Definition 1. After Step 1 of Algorithm 1, we obtain an MIS I in G, which is a dominating set of G. Throughout Step 2 and Step 3, we add a set of nodes in the path connecting each pair of nodes in I to C. Thus, $D = I \cup C$ is a CDS of G, which can satisfy Condition (i) and Condition (ii) in Definition 1.

Next, we show that D also satisfies Condition (iii) in Definition 1, which completes this proof. This condition dictates that for each pair of nodes $v_i, v_j \in V$, the nodes in a maximum spectral-efficiency path p^*_{ij} between v_i and v_j in G should be included in D. In other words, there should be a path p^D_{ij} between v_i and v_j such that $p^D_{ij} \setminus \{v_i, v_j\} \subseteq D$ and $p^D_{ij} = p^*_{ij}$, where p^*_{ij} is a maximum spectral-efficiency path between v_i and v_j in G. Since after Algorithm 1 is terminated, all nodes in G can partitioned into two subsets, I and $V \setminus I$, one of the following three cases should be true: (i) $v_i \in I$ and $v_j \in I$, (ii) $v_i \in I$ and $v_j \in V \setminus I$, and (iii) $v_i \in V \setminus I$ and $v_j \in V \setminus I$.

Case 1. $v_i \in I$ and $v_j \in I$. Let $b_i = v_i \in I$ and $b_j = v_j \in I$. Remind that in Step 2 of Algorithm 1, we compute a referential path weight $weight(b_i, b_j)^*$ for each pair of nodes $b_i, b_j \in I$. In Step 3, for each pair $b_i, b_j \in I$, we only consider the edges in E satisfying that $WL(e) \geq weight(b_i, b_j)^*$. After we remove the edges whose weight is smaller than $WL(e)$ from G, we compute the shortest path between b_i and b_j from the residual graph and store the path to P^{temp}_{ij}. We repeat the process above for each $e \in E$ with b_i and b_j fixed, and we pick the path with the largest spectral-efficiency from P^{temp}_{ij}. By Lemma 2, the path we computed in this way is a maximum spectral efficient path between b_i and b_j, i.e. $p^D_{ij} = p^*_{ij}$. Therefore, $p^*_{ij} \setminus \{b_i, b_j\} \subseteq D$.

Case 2. $v_i \in I$ and $v_j \in V \setminus I$. Let $b_i = v_i \in I$ and $v_q = v_j \in V \setminus I$. Then, since I is a dominating set of G, there has to be a node $b_q \in I$ dominating v_q. Suppose $p^*(b_i, v_q)$ is the most spectral efficient path between b_i and v_q in G. Then,

$$weight(p^*(b_i, v_q)) = \min \Big[weight(b_i, b_q)^*, WL(b_q, v_q) \Big].$$

Since $b_i \in I$ and $b_q \in I$, we have computed the following referential path weight for the nodes in Step 2:

$$weight(b_i, b_q)^* = \min \Big[weight(b_i, b_q)^{sp}, WN(b_i), WN(b_q) \Big].$$

Note that since $b_q \in I$, $WN(b_q) = \min_{v_j \in N_q} \{WL(b_q, v_j)\}$, which means $WN(b_q) \leq WL(b_q, v_q)$. Therefore, we can conclude that

$$weight(p^*(b_i, v_q)) \geq weight(b_i, b_q)^*.$$

By Lemma 2, this implies that the most spectral efficient path (shortest path) from b_i to b_q computed in the residue graph in Step 3 of Algorithm 1 is a part of the shortest path between b_i and v_q, i.e. the union of the path and the link from b_q to v_q forms the most spectral efficient path from b_i to v_q. As a result, $p^*(b_i, v_q) \setminus \{b_i, v_q\} \subseteq D$.

Case 3. $v_i \in V \setminus I$ and $v_j \in V \setminus I$. We can easily prove $p^*(v_i, v_j) \setminus \{v_i, v_j\} \subseteq D$ using the argument used in the proof of Case 2, where $p^*(v_i, v_j)$ is the most spectral efficient path between v_i and v_j in G.

From the analysis of the three cases above, we can conclude that D also satisfies Condition (iii) in Definition 1. As a result, this theorem holds true. □

Now, we introduce two interesting theorems regarding the performance of this algorithm.

Theorem 3. *Given a connected graph $G = (V, E)$, suppose D is the output of Algorithm 1 for the problem MSE-CDS. Then, $|opt_{SPCDS}| \leq |D| \leq \alpha \cdot |opt_{SPCDS}|$, where $\alpha = \max_{\forall v_i, v_j \in V} \frac{|p^*_{ij}|}{|sp(v_i, v_j)|}$, p^*_{ij} and $sp(v_i, v_j)$ are the maximum spectral efficiency routing path and the shortest path between v_i and v_j in G respectively, and opt_{SPCDS} is the optimal solution for the* shortest path connected dominating set (SPCDS) *problem in [17].*

Proof. Based on Theorem 1, we can obtain that D produced by Algorithm 1 can satisfy the third rule in Definition 1: for any pair of distinct nodes v_i and v_j in V,

$$p_{ij}^D = p_{ij}^*, \tag{8}$$

where p_{ij}^D is the path connecting v_i, v_j whose intermediate nodes all belong to D and p_{ij}^* is the maximum spectral efficiency routing path between v_i and v_j in G. Furthermore, for any pair of nodes v_i and v_j in V,

$$|sp(v_i, v_j)| \leq |p_{ij}^D| \leq \alpha' \cdot |sp(v_i, v_j)|$$

(Here we denote α' as $\max_{\forall v_i, v_j \in V} \frac{|p_{ij}^D|}{|sp(v_i,v_j)|}$, where p_{ij}^D and $sp(v_i, v_j)$ are the path with all the intermediate nodes belonging to D and the shortest path between v_i and v_j in G respectively). From Eq. (8), we can rewrite α' as $\alpha = \max_{\forall v_i, v_j \in V} \frac{|p_{ij}^*|}{|sp(v_i,v_j)|}$. Then we obtain that

$$|sp(v_i, v_j)| \leq |p_{ij}^D| \leq \alpha \cdot |sp(v_i, v_j)| \tag{9}$$

Now we recall that the SPCDS problem in [17] is formulated as follows: the SPCDS problem is to find a minimum size node set $S \subseteq V$ such that $\forall u, w \in V$ having $H(u, w) \geq 2$, $\forall p_i(u, w) = \{u, v_1, ..., v_k, w\} \in P(u, w)$, all intermediate nodes $v_1, v_2, ..., v_k$ should belong to S. Here $H(u, w)$ denotes the number of hops on the shortest path between u and w and $P(u, v)$ is a path set composed by all shortest paths between u and w.

We denotes the optimal solution obtained from the proposed algorithm in [17] as opt_{SPCDS}. Based on the above equations, the following conclusion can be easily come to based on opt_{SPCDS}: $opt_{SPCDS} \leq |D| \leq \alpha \cdot opt_{SPCDS}$. Thus, this theorem is true. □

Theorem 4. *Given a connected graph $G = (V, E)$, suppose D is the output of Algorithm 1 for the problem MSE-CDS. Without loss of generality, suppose*

$$|p^D(u_1, v_1)| \geq |p^D(u_2, v_2)|.$$

Then, for any two pairs of nodes in G, (u_1, v_1) and (u_2, v_2), $1 \leq \frac{|p^D(u_1,v_1)|}{|p^D(u_2,v_2)|} \leq \beta$, where

$$\beta = \max_{\forall v_i, v_j, v_{i'}, v_{j'} \in V, i \neq i', j \neq j'} \frac{|sp(v_i, v_j)|}{|sp(v_{i'}, v_{j'})|}$$

and $p^D(u_k, v_k)$ is the path with all the intermediate nodes belong to D between u_k and v_k in G ($k = 1, 2$).

Proof. For any two pairs of nodes in G, (u_1, v_1) and (u_2, v_2) (without loss of generality, suppose that $|p^D(u_1, v_1)| \geq |p^D(u_2, v_2)|$),

$$\frac{|p^D(u_1, v_1)|}{|p^D(u_2, v_2)|} \leq \max_{p^D_{ij}, p^D_{i'j'} \in DP, i \neq i', j \neq j'} \frac{|p^D_{ij}|}{|p^D_{i'j'}|} = \frac{|p^D_{max}|}{|p^D_{min}|}, \tag{10}$$

where $p^D_{max} = \arg\max_{p^D_{i''j''} \in DP} |p^D_{i''j''}|$ and $p^D_{min} = \arg\min_{p^D_{i''j''} \in DP} |p^D_{i''j''}|$. From the analysis of the proof of Theorem 3, we find that $|p^D_{ij}| \leq \alpha \cdot |sp(v_i, v_j)|$ based on Eq. (9). Thus, we have that

$$\begin{aligned} &|p^D_{max}| \leq \alpha \cdot |sp(max_i, max_j)| \text{ and} \\ &|p^D_{min}| \leq \alpha \cdot |sp(min_i, min_j)|, \end{aligned} \tag{11}$$

$$\text{where } \alpha = \max_{\forall v_i, v_j \in V} \frac{|p^*_{ij}|}{|sp(v_i, v_j)|},$$

and $sp(max_i, max_j)$ and $sp(min_i, min_j)$ are the shortest paths between the two ends of p^D_{max} and between the two ends of p^D_{min} respectively. From Eq. (10) and Eq. (11), we can conclude

$$\begin{aligned} \frac{|p^D(u_1, v_1)|}{|p^D(u_2, v_2)|} &\leq \frac{|sp(max_i, max_j)|}{|sp(min_i, min_j)|} \\ &\leq \max_{\forall v_i, v_j, v_{i'}, v_{j'} \in V, i \neq i', j \neq j'} \frac{|sp(v_i, v_j)|}{|sp(v_{i'}, v_{j'})|}. \end{aligned}$$

Then we can finally obtain that

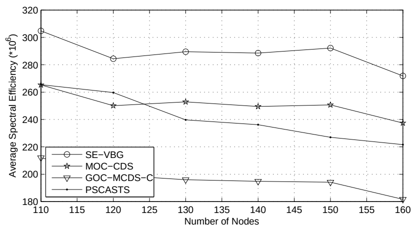
$$1 \leq \frac{|p^D(u_1, v_1)|}{|p^D(u_2, v_2)|} \leq \beta, \text{ where}$$

$$\beta = \max_{\forall v_i, v_j, v_{i'}, v_{j'} \in V, i \neq i', j \neq j'} \frac{|sp(v_i, v_j)|}{|sp(v_{i'}, v_{j'})|},$$

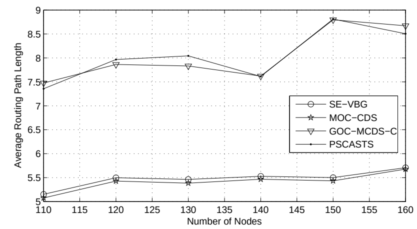
which completes the proof of this theorem. □

6. Simulation Results

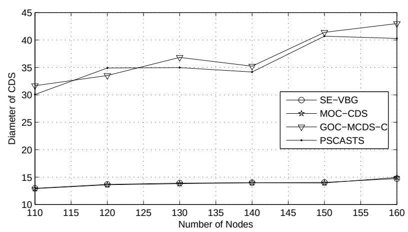
In this section, we compare the average performance of our algorithm, SE-VBG, against the following three algorithms.



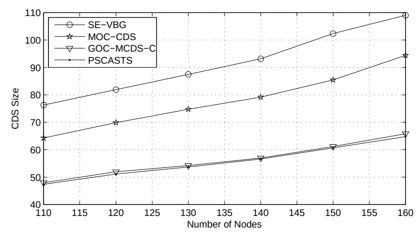
(a) Comparison of average CDS spectral-efficiency



(b) Comparison of average routing path length



(c) Comparison of diameter



(d) Comparison of CDS size

Figure 1: Average performance comparison of SE-VBG, MOC-CDS [16], GOC-MCDS-C [20], and PSCASTS [21].

- MOC-CDS [16]: its goal is to compute a smaller size CDS including a shortest path between every pair of nodes in the original graph. MOC-CDS generates CDS whose size is at most a constant factor far from an optimal solution.

- GOC-MCDS-C [20]: its goal is to solve a similar problem tackled by MOC-CDS. Unlike MOC-CDS, GOC-MCDS-C has a constant factor performance ratio in terms of the size as well as the stretch factor of CDS.

- PSCASTS [21]: its sole goal is to produce a CDS with smaller size. Das *et al.* showed that PSCASTS outperforms all of the existing algorithms in terms of the size of CDS on average.

In this simulation, we first set the size of a virtual space as 10 meters by 10 meters and randomly deploy a set of nodes whose number varies from 110 to 160. We assume the transmission range of each node is 1 meter. In case a network instance is disconnected, we discard it and produce another one until we obtain a connected network. Per each parameter setting, we

Algorithm	Msg Sent	Msg Rcv'd	Ratio
SE-VBG	36758	944	2.56815%
MOC-CDS	36758	896	2.43756%
GOC-MCDS-C	36758	896	2.43756 %
PSCASTS	36758	855	2.32602%

Table 1: Our result shows the throughput (the amount of packets delivered during a fixed period) of SE-VBG is the highest.

produce 30 graph instances and execute each of the algorithms. For each CDS generated, we measure the quality of it in terms of the following four metrics, average spectral-efficiency (the average of spectral-efficiency of the most spectral-efficient path over the CDS between every pair of nodes), average routing path length (the average of shortest path length over the CDS between every pair of nodes), diameter (the longest path length over the CDS between a pair of nodes), and size (the cardinality of CDS). To evaluate the SNR, we follow the normalized parameter setting from [28], and set $\alpha = 3, B = 1024, P = 100, N_0 = 100$, and $c = 2$. Fig. 1 illustrates the simulation results. Fig. 1(a) shows the average spectral-efficiency of CDSs produced by SE-VBG is much higher than the other algorithms. Note that based on the results in [28], a path with higher spectral-efficiency than shortest hop length has higher throughput. Therefore, the throughput of CDS produced by SE-VBG is the highest on average. Still, Fig. 1(b) and Fig. 1(c) show that the diameter and the average routing path length of CDS generated by our algorithm are very competitive. Meanwhile, Fig. 1(d) shows the average size of CDS produced by SE-VBG is larger than CDSs produced by the other competitors. This indicates that an algorithm trying to produce a CDS with minimum cardinality may generate a CDS whose throughput is low as implicitly concerned by many other previous work [16, 20]. After all, the results also indicate our algorithm produces a CDS with higher communication efficiency in exchange of its size.

Finally, we conduct another simulation using OMNet++ to see which algorithm produces a CDS with higher throughput. In this simulation, we set the size of virtual space to 280 meters by 280 meters and randomly deploy 50 nodes. We assume that each node has a probability of sending a uniform length packet of 70 % per every 0.5 second to a random receiver. We continue this simulation for 400 seconds and see how many packets are actually delivered. We also set the transmission power of each node is 50mW, and the thermal noise is -85 dBm. Table 1 shows the result of this simulation. Our result indicate that the CDS produced by SE-VBG has a higher spectral-efficiency as well as higher throughput. Note that due to the smaller size, the throughput of the CDS produced by PSCASTS is the lowest.

We would like to remind that the actual motivation of introducing the concept of virtual backbone is to boost the performance of wireless networks and several metrics such as the size of virtual backbone and the average hop distance have served as abstract metric. However, our algorithm outperforms the other competitors in terms of throughput, which is a performance metric of real importance, and therefore, the fact that our algorithm produces a CDS whose size is slightly larger than that produced by MOC-CDS on average does not undermine the importance of our contribution significantly.

7. Concluding Remarks

In the literature, there are several arguments why virtual backbone should be employed in wireless networks. The most widely used argument is that by reducing the number of nodes involved in, we can reduce the amount of wireless signal collision and interference as well as reduce burden of managing routing related information, and therefore the network becomes more efficient. Apparently, those benefits can be maximized by adopting a virtual backbone with smaller size. However, as claimed in the other works, a minimal size virtual backbone could cause communication inefficiency since any two nodes may need to communicate with each other through a longer path provided by the backbone.

In this paper, we tried to look at the one significant aspect of this problem which has been ignored so far, throughput. Since the virtual backbone is a kind of common communication utility in wireless networks, throughput is clearly an important metric. This motivated us to investigate the possibility of applying a new metric, spectral-efficiency, in the design of a connected dominating set computation algorithm in unit disk graph. In summary, there are several metrics to be considered in the design of virtual backbone construction algorithms. Especially, the size of virtual backbone seems to have a trade-off relationship with the other metrics such as routing cost and throughput. We believe that adopting a minimum size virtual backbone is not always beneficial when we are trying to maximize the throughput. However, what is the best balance between them to maximize the benefit of virtual backbone should be further investigated.

Acknowledgment

This paper was jointly supported by National Natural Science Foundation of China under grant 91124001, the Fundamental Research Funds for the Central Universities, and the Research Funds of Renmin University of China 10XNJ032. It was also supported in part by US National Science Foundation (NSF) CREST No. HRD-1345219.

References

[1] M.X. Gong, D. Akhmetov, R. Want, and Shiwen Mao, "Directional CSMA/CA Protocol with Spatial Reuse for mmWave Wireless Networks," *roceedings of IEEE Global Communication Conference (Globecom)*, 2010.

[2] M. Chen, T. Kwon, S. Mao, V.C.M. Leung, "Spatial Temporal Relation-based Energy-efficient Reliable Routing Protocol in Wireless Sensor Networks," International Journal of Sensor Networks (IJSN), vol. 5, issue 3, pp. 129-141, July 2009.

[3] S.-Y. Ni, Y.-C. Tseng, Y.-S. Chen, and J.-P. Sheu, "The Broadcast Storm Problem in a Mobile Ad Hoc Network," *Proceedings of the 5th Annual ACM/IEEE International Conference on Mobile Computing and Networking (MobiCom)*, pp. 151-162, 1999.

[4] A. Ephremides, J.E. Wieselthier, and D.J. Baker, "A Design Concept for Reliable Mobile Radio Networks with Frequency Hopping Signaling," *Proceedings of the IEEE*, vol. 75, no. 1, pp. 56-73, 1987.

[5] S. Guha and S. Khuller, "Approximation Algorithms for Connected Dominating Sets," *Algorithmica*, vol. 20, pp. 374-387, 1996.

[6] L. Ruan, H. Du, X. Jia, W. Wu, Y. Li, and K.-I Ko, "A Greedy Approximation for Minimum Connected Dominating Sets," *Theoretical Computer Science (TCS)*, vol. 329, pp. 325-330, 2004.

[7] X. Cheng, X. Huang, D. Li, W. Wu, and D.-Z. Du, "Polynomial-time Approximation Scheme for Minimum Connected Dominating Set in Ad Hoc Wireless Networks," *Networks*, vol. 42, 2003.

[8] P.-J. Wan, K.M. Alzoubi, and O. Frieder, "Distributed Construction of Connected Dominating Set in Wireless Ad Hoc Networks," *Mobile Networks and Applications - Discrete Algorithms and Methods for Mobile Computing and Communications*, vol. 9, no. 2, pp. 141-149, 2004.

[9] S. Funke, A. Kesselman, U. Meyer, and M. Segal, "A Simple Improved Distributed Algorithm for Minimum CDS in Unit Disk Graphs," *ACM Transactions on Sensor Networks (TOSN)*, vol. 2, no. 3, pp. 444-453, 2006.

[10] X. Gao, Y. Wang, X. Li, and W. Wu, "Analysis on Theoretical Bounds for Approximating Dominating Set Problems," *Discrete Mathematics, Algorithms and Applications (DMAA)*, vol. 1, no. 1, pp. 71-84, 2009.

[11] P.-J. Wan, L. Wang, and F. Yao, "Two-phased Approximation Algorithms for Minimum CDS in Wireless Ad Hoc Networks," *Proceedings of the 2008 The 28th International Conference on Distributed Computing Systems (ICDCS)*, pp. 337–344, 2008.

[12] M. Li, P.-J. Wan, and F. Yao, "Tighter Approximation Bounds for Minimum CDS in Wireless Ad Hoc Networks," *Proceedings of the 20th International Symposium on Algorithms and Computation (ISAAC)*, pp. 699-709, 2009.

[13] W. Wu, H. Du, X. Jia, Y. Li, and S.C.-H. Huang, "Minimum Cnnected Dominating Sets and Maximal Independent Sets in Unit Disk Graphs," *Theoretical Computer Science (TCS)*, vol. 352, no. 1, pp. 1-7, 2006.

[14] M.T. Thai, R. Tiwari, and D.-Z. Du, "On Construction of Virtual Backbone in Wireless Ad Hoc Networks with Unidirectional Links," *IEEE Transactions on Mobile Computing (TMC)*, vol. 7, no. 9, pp. 1098-1109, 2008.

[15] M.T. Thai, F. Wang, D. Liu, S. Zhu, and D.-Z. Du, "Connected Dominating Sets in Wireless Networks with Different Transmission Ranges," *IEEE Transactions on Mobile Computing (TMC)*, vol. 6, no. 7, pp. 721-730, 2007.

[16] L. Ding, X. Gao, W. Wu, W. Lee, X. Zhu, and D.-Z. Du, "Distributed Construction of Connected Dominating Sets with Minimum Routing

Cost in Wireless Networks," *2010 IEEE 30th International Conference on Distributed Computing Systems (ICDCS)*, pp. 448-457, 2010.

[17] L. Ding, X. Gao, W. Wu, W. Lee, X. Zhu, and D.-Z. Du, "An Exact Algorithm for Minimum CDS with Shortest Path Constraint in Wireless Networks," *Optimization Letters*, vol. 5, no. 2, pp. 297-306, 2011.

[18] D. Kim, Y. Wu, Y. Li, F. Zou, and D.-Z. Du, "Constructing Minimum Connected Dominating Sets with Bounded Diameters in Wireless Networks," *IEEE Transactions on Parallel and Distributed Systems (TPDS)*, vol. 20, no. 2, pp. 147-157, 2009.

[19] L. Ding, W. Wu, J. Willson, H. Du, W. Lee, and D.-Z. Du, "Efficient Algorithms for Topology Control Problem with Routing Cost Constraints in Wireless Networks," *IEEE Transactions on Parallel and Distributed Systems (TPDS)*, vol. 22, no. 10, pp. 1601-1609, 2011.

[20] H. Du, W. Wu, Q. Ye, D. Li, W. Lee, and X. Xu, "CDS-based Virtual Backbone Construction with Guaranteed Routing Cost in Wireless Sensor Networks," *IEEE Transactions on Parallel and Distributed Systems (TPDS)*, vol. 24, issue 4, pp. 652-661, 2013.

[21] A. Das, C. Mandal, C. Reade, and M. Aasawat, "An Improved Greedy Construction of Minimum Connected Dominating Sets in Wireless Networks," *Wireless Communications and Networking Conference (WCNC)*, pp. 790-795, 2011.

[22] P. Gupta and P.R. Kumar, "The Capacity of Wireless Networks," *IEEE Transactions on Information Theory (TIT)*, vol. 46, no. 2, pp. 388-404, 2000.

[23] O. Oyman and S. Sandhu, "A Shannon-Theoretic Perspective on Fading Multihop Networks," *Proceedings of the 40th Annual Conference on Information Sciences and Systems*, pp. 525-530, 2006.

[24] F. Xue, L.-L. Xie, and P.R. Kumar, "The Transport Capacity of Wireless Networks over Fading Channels," *IEEE Transactions on Information Theory (TIT)*, vol. 51, pp. 834-847, 2005.

[25] D. Couto, S.J. Douglas, D. Aguayo, J. Bicket, and R. Morris, "A High-throughput Path Metric for Multi-hop Wireless Routing," *Proceedings*

of the 9th Annual International Conference on Mobile Computing and Networking (MobiCom), pp. 134-146, 2003.

[26] R. Draves, J. Padhye, and B. Zill, "Routing in Multi-radio, Multi-hop Wireless Mesh Networks," *Proceedings of the 10th Annual International Conference on Mobile Computing and Networking (MobiCom)*, pp. 114-128, 2004.

[27] X. Cheng, D.-Z. Du, L. Wang, and B. Xu, "Relay Sensor Placement in Wireless Sensor Networks," *Wireless Networks*, vol. 14, no. 3, pp. 347-355, 2008.

[28] D. Chen, M. Haenggi, and J.N. Laneman, "Distributed Spectrum-efficient Routing Algorithms in Wireless Networks," *IEEE Transactions on Wireless Communications (TWC)*, vol. 7, no. 12, 2008.

[29] M. Saad, "Optimal Spectrum-efficient Routing in Multihop Wireless Networks," *IEEE Transactions on Wirless Communications (TWC)*, vol. 8, no. 12, pp. 5822-5826, 2009.

[30] L. Xiao, M. Johansson, and S. Boyd, "Simultaneous Routing and Resource Allocation via Dual Decomposition," *IEEE Transactions on Communications (TC)*, vol. 52, pp. 1136-1144, 2002.

[31] M. Sikora, J.N. Laneman, M. Haenggi, D.J. Costello, and T.E. Fuja, "Bandwidth- and Power-efficient Routing in Linear Wireless Networks," *IEEE Transactions on Information Theory (TIT)*, vol. 52, no. 6, pp. 2624-2633, 2006.

[32] O. Oyman and S. Sandhu, "A Shannon-Theoretic Perspective on Fading Multihop Networks," *Proceedings of the 40th Annual Conference on Information Sciences and Systems (CISS)*, pp. 525-530, 2006.

[33] O. Oyman and S. Sandhu, "Non-Ergodic Power-Bandwidth Tradeoff in Linear Multi-hop Networks," *Proceedings of IEEE International Symposium on Information Theory*, pp. 1514-1518, 2006.

[34] M. Gastpar and M. Vetterli, "On the Capacity of Large Gaussian Relay Networks," *IEEE Transactions on Information Theory (TIT)*, vol. 51, no. 3, pp. 765-779, 2005.

[35] T.S. Rappaport, "Wireless Communications: Principles and Practice," *IEEE Press*, 1996.

[36] M. Haenggi and D. Puccinelli, "Routing in Ad Hoc Networks: a Case for Long Hops," *IEEE Communications Magazine*, vol. 43, no. 10, pp. 93-101, 2005.

[37] T.M. Cover and J.A. Thomas, "Elements of Information Theory," *Wiley-Interscience*, 1991.