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Computing an Effective Decision Making Group of a Society using Social Network Analysis

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Abstract Recent years have witnessed how much a decision making group can be dysfunctional due to the extreme hyperpartisanship. While partisanship is crucial for the representatives to pursue the wishes of those whom they represent for, such an extremism results in a severe gridlock in the decision making progress, and makes themselves highly inefficient. It is known that such a problem can be mitigated by having negotiators in the group. This paper investigates the potential of social network analysis techniques to choose an effective leadership group of a society such that it suffers less from the extreme hyperpartisanship. We establish three essential requirements for an effective representative group, namely Influenceability, Partisanship, and Bipartisanship. Then, we formulate the problem of finding a minimum size representative group satisfying the three requirements as the minimum connected k-core dominating set problem (MCkCDSP), and show its NP-hardness. We introduce an extension of MCkCDSP, namely MCkCDSP-C, which assumes the society has a number of sub-communities and requires at least one rep-

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resentative from each sub-community should be in the leadership. We also propose an approximation algorithm for a subclass of MCkCDSP with $k = 2$, and show an α-approximation algorithm of MCkCDSP can be used to obtain an α-approximation algorithm of MCkCDSP-SC.

Keywords Dominating set · social networks · approximation algorithm · k-core · vertex connectivity

1 Introduction

In our daily lives, the way we think and behave is continuously affected by the opinions of our family members and close friends. Therefore, by studying the relationship among individuals in our society, we can extract information which is potentially very useful for important decision making processes. Due to the reason, the study of social networks, which are usually represented by graphs, has been received intensive attentions recently. In a social network graph, each node represents a member of the society. Also, there is an edge from one to another if the former has an influence to the later. Depending on the context, the edge can be either directional or bidirectional. Given a graph, a *dominating set (DS)* is a subset of the nodes in the graph such that all other nodes are adjacent to at least one node in the subset. In the context of social science, a DS of a social network graph means a subgroup which has a relationship with the rest. In the real world, such a subgroup tends to be highly influential to the whole society. Usually, the efficiency and the significance of the subgroup increases as its size decreases.

Due to the crucialness of the implication of the DS in the field of social science and related fields, DS has received lot of attention [2–7]. For instance, recently, Feng et al. [4] and Wang et al. [5] are independently studied the problem of computing minimum cardinality DS of a given social network graph such that each node u outside the subset has at least $\lceil \frac{deg(u)}{2} \rceil$ neighboring nodes in the subset, where $deg(u)$ is the degree of u in the graph (fast information propagation problem [4] and minimum *positive influence dominating set (PIDS)* problem [5], respectively). This problem is important since using the subset, we can efficiently spread ideas and information within a group (i.e. advertisement). In [6], Dinh et al. introduced the concept of *total positive influence dominating set (TPIDS)*, which is a more generalized version of PIDS. So far, most of DS related problems in social networking have focused on the relationship between the DS nodes and the rest. However, only few works have been done in which the relationship among the DS nodes, which could be computationally very interesting, are also used to solve issues in social networking. To the best of our knowledge, only [7] considers a minimum PIDS computation problem with connectivity requirement.

Recent years have witnessed how much a decision making group can be dysfunctional due to the extreme hyperpartisanship. While partisanship is crucial for the representatives to pursue the wishes of those whom they represent for, such an extremism results in a severe gridlock in the decision making progress, and makes themselves highly inefficient [8]. It is suggested that such a problem can be mitigated by having negotiators in the group [9]. In this paper, we investigate the potential of social network analysis techniques to choose an effective leadership group of a society such that it suffers less from the extreme hyperpartisanship. We establish three essential requirements for an effective representative group, namely Influenceability, Partisanship, and Bipartisanship as below. It is clear that by making the size of such group as small as possible, the group will become more efficient. As a result, we formulate a new minimum size DS problem, which requires a special property among the members of the DS.

(a) **Bipartisanship**: for any two members in the group, there always have to be mediators between them if they are not direct collaborators.
(b) **Partisanship**: each leader should have a sufficient number of collaborators in the group. By being together with political collaborators, a leader will certainly maintain its belief than by being alone.
(c) **Influenceability**: the leader group should have a good influence over the rest of the members of the society. This property has been considered as the most important quality of a leader in many leadership quality studies.

Contributions. We model the problem of selecting a efficient leader group of a given social network graph as the *minimum connected k-core dominating set problem (MCkCDSP)*. We would like to emphasize that this is one of the few attempts to study a variation of the minimum DS problem with constraints on the relationship among the dominating nodes. We prove the problem is NP-hard and propose an approximation algorithm for a subclass of MCkCDSP with $k = 2$. In addition, we introduce an extension of MCkCDSP, namely MCkCDSP-C, which assumes the society has a number of communities and at least one representative from each community should be in the leader group. Finally, we show an α-approximation algorithm of MCkCDSP can be used to obtain an α-approximation algorithm of MCkCDSP-C.

Organizations. The rest of this paper is organized as follows. Section 2 introduces several important notations and definitions. We introduce our approximation algorithm for MCkCDSP in Section 3. A variation of MCkCDSP, namely MCkCDSP-C, and our strategy to obtain an approximation of MCkCDSP-C is in Section 4. Finally, we conclude this paper in Section 5.

2 Terms, Notations, and Definitions

In our work, we will use a directional social network graph to present a given society, in which there is a directional edge from a node u to another node v if u has an influence over v. As a result, $G = (V, E)$ represents a social network graph with node set V and directional edge set E. For any subset $D \subseteq V$, $G[D]$ is a subgraph of G induced by D. Now, let us introduce several important definitions.

Definition 1 (k-core) Given a graph $G = (V, E)$ and a positive integer k, a subset $D \subseteq V$ is a *k-core* if for each node $u \in D$, u is bidirectionally neighboring with at least k other nodes in $D \setminus \{u\}$.

Note that a subgraph of G induced by a k-core D of a given graph G is not necessarily bidirectionally connected and can be even disconnected.

Definition 2 (Connected k-core) Given a graph $G = (V, E)$ and a positive integer k, a subset $D \subseteq V$ is a *connected k-core* if (a) D is a k-core and (b) $G[D]$ is connected via bidirectional edges.

Definition 3 (Connected k-core DS) Given a graph $G = (V, E)$ and a positive integer k, a subset $D \subseteq V$ is a *connected-k-core dominating set (CkCDS)* of G if (a) D is a DS of G, and (b) D is a connected-k-core.

Definition 4 (MCkCDSP) Given a graph $G = (V, E)$ and a positive integer k, the minimum CkCDS problem (MCkCDSP) is to find a CkCDS of G with minimum cardinality.

Lemma 1 *Consider a graph $G = (V, E)$ with bidirectional edges only. Then, a subset $D \subseteq V$ is a connected-1-core if and only if $G[D]$ is connected.*

Proof First, if $G[D]$ is connected, then for each $u \in D$, $\exists v \in D \setminus \{v\}$ such that there is a link between u and v. Therefore, D is a 1-core. Reversely, by the definition of the connected k-core presented above, the subgraph of G induced by a connected-1-core in G is connected. As a result, this lemma is true.

Theorem 1 *MkCDSP is NP-hard.*

Proof We show MCkCDSP is NP-hard by showing its special case, in which $k = 1$ and all edges in G are bidirectional, is NP-hard. In this special case, given a graph $G = (V, E)$, a subset $D \subseteq V$ is a feasible solution of MCkCDSP if (a) D is a DS of G, and (b) D is a connected-1-core. By Lemma 1, the second condition is equivalent to require D to be connected. As a result, in this special case, MCkCDSP is equivalent to the *famous minimum connected dominating set (CDS)* problem in general graphs, which is a well-known NP-hard problem [10]. As a result, MCkCDSP is NP-hard and this theorem is correct.

3 A New Approximation Algorithm for MC2CDSP

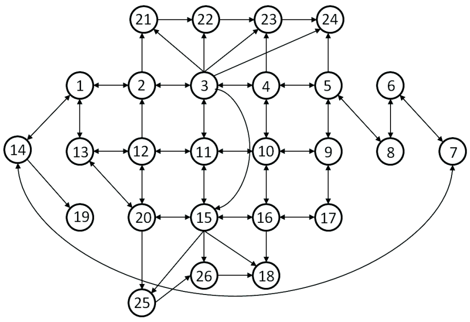
In this section, we propose the *minimum connected 2-core dominating set algorithm (MC2CDSA)*, an approximation algorithm for MC2CDSP, a special case of MCkCDSP with $k = 2$. We assume that G contains a feasible solution of the problem. Given a graph $G = (V, E)$, suppose $G' = (V', E')$ is a subgraph of G that we can obtain after we remove all directional edges from G. Note that if G includes a feasible solution of MC2CDSP, (a) there exists only one G', and (b) for each node $u \in V \setminus V'$, there exists $v \in V'$ such that $(v, u) \in E$,

Algorithm 1 MC2CDSA $(G = (V, E), k = 2)$

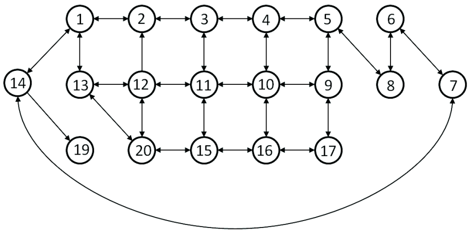
1: /* Phase 1 starts here. */
2: Build $G' = (V', E')$ such that $V' \leftarrow V$ and E' has only bidirectional edges in E.
3: Apply Guha and Khuller's 2 stage greedy algorithm [11] to G' and obtain a CDS D.
4: Color all nodes in D in white.
5: For each white node $u \in D$, color u in black if $\exists v, w \in D \setminus \{u\}$ such that u is connected to v and w via bidirectional edges, respectively.
6: **while** there is a white node $u \in D$ **do**
7:     Find a path $P \subset V' \setminus D$ with length at most 3 hops from u to another node $v \in D \setminus \{u\}$. Set $D \leftarrow D \bigcup P$, and color u, v and all nodes in P in black.
8: **end while**
9: /* Phase 2 starts here. */
10: Construct a bipartite graph $B = (V_L, V_R, E_B)$ such that $V_L \leftarrow V' \setminus D$, $V_R \leftarrow V \setminus V'$, and for each pair of $u \in V_L$ and $v \in V_R$, $E_B \leftarrow (u, v)$, a directional edge from u to v only if $(u, v) \in E$.
11: Apply a set-cover greedy algorithm on B and obtain a subset $L \subseteq V_L$.
12: **for** each node in $w \in L$ **do**
13:     **if** w is connected to only one node in D in G' **then**
14:         Find a node in $u \in L$, which is neighboring to w in G'.
15:         Add u and w to D.
16:     **end if**
17: **end for**
18: Output $D \bigcup L$.

i.e. there exists a directional edge from v to u. Largely, MC2CDSP consists of two phases. In the first phase (Lines 1-8 of Algorithm 1, see Fig. 1) for illustration), MC2CDSP adopts Guha and Khuller's 2 stage $(3 + \ln \Delta)$-approximation algorithm [11] for the minimum *connected dominating set (CDS)* problem to G' and obtain a CDS D of G'. Next, we initially color all the nodes in D in white, but we color those nodes having at least two neighbors in D in black. Then, until no white node left in D, we repeatedly (a) pick a white node $u \in D$, and find a path P in G' with length at most 3 hops from u to another node $v \in D \setminus \{u\}$ such that $P \bigcap D = \emptyset$, and (b) add the nodes in P to D. Color u, v, and all nodes in P in black.

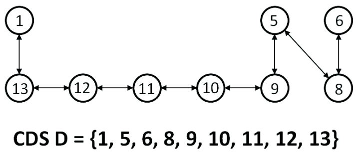
Once the first phase is done, D becomes a connected-2-core. However, it is possible that there exist some nodes in $V \setminus V'$ not dominated by D. In the second phase of the algorithm (Lines 9-18, see Fig. 2 for illustration), this problem is resolved. In detail, to make D to be a DS of G, we need to add more nodes from V' to D such that D is still a connected-2-core and all nodes in $V \setminus V'$ are dominated by some nodes in D. For this purpose, we first construct a bipartite graph $B = (V_L, V_R, E_B)$ such that V_L is the set of nodes in $V' \setminus D$, V_R is the set of nodes in $V \setminus V'$. At last, establish an edge from each node $u \in V_L$ to another node $v \in V_R$ in E_B only if there exists a directional edge from u to v in G. Then, we apply a well-known set-cover greedy approximation algorithm with performance ratio $H(\Delta)$ on B, where H is a harmonic function. Once a subset L of V_L is selected by the algorithm, for each node in $w \in L$, we check if w is bidirectionally connected to another node $w' \in L$ in G. Otherwise, we select another node in $V_L \setminus L$ such that $D \bigcup L$



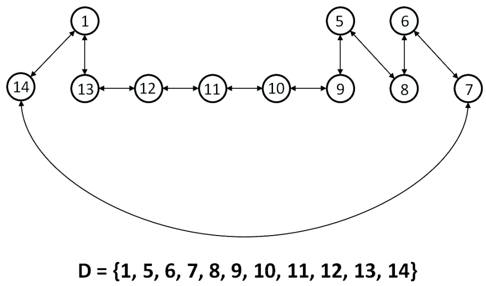
(a) An input graph $G = (V, E)$.



(b) $G' = (V', E')$ is constructed, where $V' \leftarrow V$ and E' is the set of bidirectional edges in E.

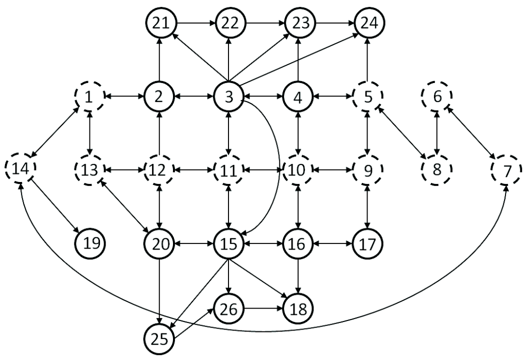


(c) Computation of a connected dominating set of G'.

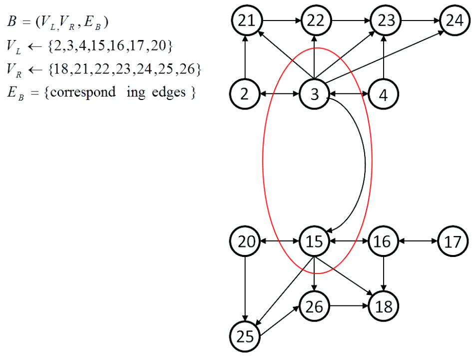


(d) Computation of a connected 2-core dominating set of G'.

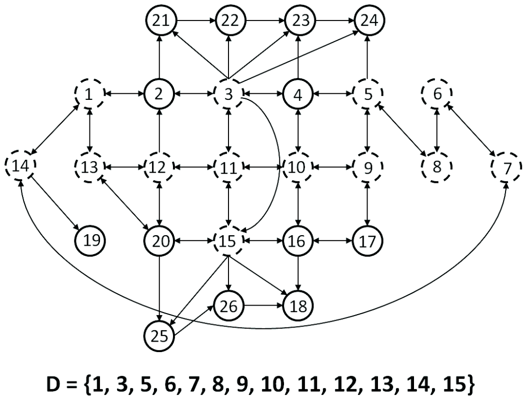
Fig. 1 Example of Phase 1 of MC2CDSA.



(a) Some nodes in G may not be dominated by the connected 2-core dominating set of G'.



(b) Add some nodes to the existing connected 2-core dominating set using the set cover greedy strategy.



(c) Final connected 2-core dominating set of G.

Fig. 2 Example of Phase 2 of MC2CDSA.

remains to be a connected-2-core. As a result, $D \bigcup L$ is a feasible solution of the MC2CDSP instance.

Theorem 2 *The output of MC2CDSA is a feasible solution of MC2CDSP.*

Proof From Lines 2-3, we obtain a CDS D of G'. Then, each node $u \in D$ has to have at least one or more neighbors in D since $G'[D]$ is connected. Suppose u has only one neighbor in $G'[D]$. Then, there has to be another node $v \in V' \setminus D$ connected to u in G'. Otherwise, u has no neighbor in G', which implies that G has no feasible solution. Also, notice that v has to be adjacent at least one neighbor in $G'[D]$. Therefore, if we add u and v to D, then each of u and v has at least two neighbors in D. By repeating this process for all nodes like u in D, D will eventually become a connected 2-core. Clearly, each node $u \in V' \setminus D$ has to be adjacent to at least one node $v \in D$, otherwise, D is not a dominating set of G'. However, there exist some nodes in $V \setminus V'$ which are not dominated by D. To dominate them, we need to add more nodes from $V' \setminus D$ to D. The minimum number of the nodes can be approximated by solving the set-cover problem over the biaprtite graph B induced in Line 10. Unfortunately, once the set L is constructed by the algorithm, $D \cup L$ is not necessarily a 2-core even though it is a connected 1-core dominating set of G. To ensure the 2-core-ness of $D \cup L$, we need to add more nodes for those in L. (Line 13-17). In detail, for each node $u \in L$, if it has only neighbor in D, find another node $v \in V' \setminus (L \bigcup D)$ connected to u. We can prove such a v does exists all the time as long as there is a feasible solution of the problem in G using the similar argument given above. As a result, $L \bigcup D$ is a connected 2-core dominating set of G, and this theorem is true.

Theorem 3 *The performance ratio of MC2CDSA for MC2CDSP is* $3 \cdot (1 + 2\Delta) \cdot (3 + \ln \Delta)$.

Proof Suppose OPT is an optimal solution of MC2CDSP. In the first phase, we adopt a Guha and Khuller's 2 stage $(3+\ln \Delta)$-approximation algorithm [11] to obtain a CDS D of G'. As we proved in Theorem 1, a CDS of G' is also a connected 1-core, we have $|D| \leq (3+\ln \Delta)|OPT|$. Next, to make D a connected 2-core, we add a series of paths with length at most three. Therefore, for each node in D, we add at most 2 more nodes. As a result, we have

$$|D| \leq (3 + \ln \Delta)|OPT| + 2 \cdot (3 + \ln \Delta)|OPT| = 3 \cdot (3 + \ln \Delta)|OPT|.$$

In the second phase, we add more nodes D so that D can dominate the nodes in $V \setminus V'$. Note that the size of L is bounded by the size of D constructed so far multiplied by the maximum degree of G', Δ, and thus we have, $|L| \leq \Delta \cdot 3 \cdot (3 + \ln \Delta)|OPT|$. Furthermore, for each node in L, we may need to add one more nodes to make $L \cup D$ a 2-core. Therefore, we have

$$|D \cup L| \leq 3 \cdot (3+\ln \Delta)|OPT| + (1+1)\Delta \cdot 3 \cdot (3+\ln \Delta)|OPT| = 3 \cdot (1+2\Delta) \cdot (3+\ln \Delta)|OPT|,$$

and thus this theorem is true.

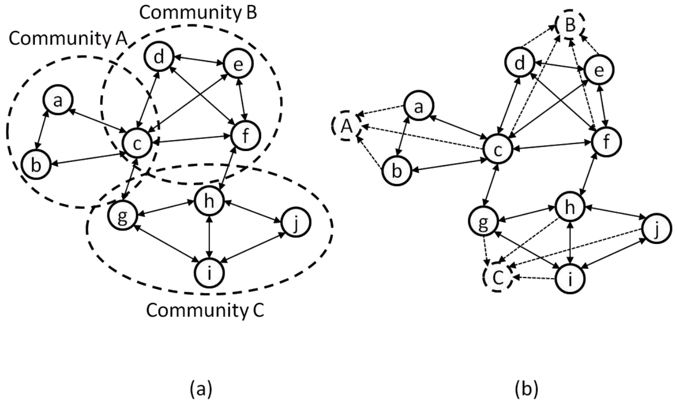


Fig. 3 These figures illustrate an graph conversion example from MCkCDSP-C problem instance (Fig.(a)) to MCkCDSP problem instance (Fig.(b)).

4 MCkCDSP under Sub-community Structures

Previously, we introduced MCkCDSP whose goal is to find a connected k-core dominating set of a given social network. In the real application domain, the objective of this problem is to elect a representative group which can effectively operate. However, the formulation ignores one important aspect of our social situation that there are various underlying sub-community structures which are abundant in the society. Suppose we want to solve MCkCDSP in a way that at least one representative from each sub-community is included in the representative group, which is likely to happen in real world situation. Then, any algorithm for MCkCDSA is not useful anymore. Therefore, in this section, we introduce a variation of MCkCDSP, to deal with this new challenge. We first introduce a variation of MCkCDSP to formulate this problem and show how it can be solved using our result so far.

Definition 5 (MCkCDSP-C) Given a directional social network graph $G = (V, E)$, a collection $\mathcal{C} = \{C_1, C_2, \cdots, C_l\}$ of the subsets of V, and a positive integer k, the minimum CkCDS problem with communities (MCkCDSP-C) is to find a CkCDS D of G with minimum cardinality such that for each subset $C_i \in \mathcal{C}$, $C_i \bigcup D \neq \emptyset$.

Corollary 1 *MCkCDSP-C is NP-hard.*

Proof The proof of this corollary naturally follows from Theorem 1 since MCkCDSP-C with $\mathcal{C} = \emptyset$ is equivalent to MCkCDSP.

Given an MCkCDSP-C instance $\langle G = (V, E), \mathcal{C}, k \rangle$, we first induce a graph $G' = (V', E')$ from a given MCkCDSP instance $\langle G = (V, E), \mathcal{C}, k \rangle$ as follow.

(a) Copy G to G', i.e. $V' \leftarrow V$ and $E' \leftarrow E$.

(b) For each subset $C_i \in \mathcal{C}$, add a node c_i to V', and add an edge from each node $u \in C_i$ to c_i to E'.

One example of this graph induction is shown in Fig. 3, e.g. G is in Fig. 3(a) and G' is in Fig. 3(b). Then, we have the following lemma.

Theorem 4 *There exists a feasible solution of an MCkCDSP-C instance $\langle G, \mathcal{C}, k\rangle$ if and only if there exists a feasible solution of an MCkCDSP instance $\langle G', k\rangle$.*

Proof We first show that a feasible solution of an MCkCDSP-C instance $\langle G, \mathcal{C}, k\rangle$ is a feasible solution of an MCkCDSP instance $\langle G', k\rangle$. Suppose D is a feasible solution of the MCkCDSP-C instance $\langle G, \mathcal{C}, k\rangle$. Then, D is clearly a connected k-core and dominating all nodes in V in G, which implies that D is a connected k-core and dominating all nodes in $V' \setminus \{c_1, \cdots, c_l\}$. By the definition of MCkCDSP-C, for each $C_i \in \mathcal{C}$, at least one node in C_i is included in D. In addition, by the construction of G', each c_i is dominated by all nodes in $C_i \subset V'$. Therefore, for each c_i, there exists at least one node in D dominating c_i in G'. Next, we show that a feasible solution of an MCkCDSP instance $\langle G', k\rangle$ is a feasible solution of an MCkCDSP-C instance $\langle G, \mathcal{C}, k\rangle$. Suppose Suppose D' is a feasible solution of the MCkCDSP instance $\langle G', k\rangle$. Then, for each c_i, there exists a node $u \in D'$ dominating c_i by the definition of MCkCDSP. This means that for each $C_i \in \mathcal{C}$, there exists a node from C_i in D' in G by the construction of G'. Furthermore, D' is a connected k-core and dominating all nodes in V', which means that D' is dominating all nodes in V. As a result, this theorem is true.

Theorem 5 *There exists an α-approximation algorithm for MCkCDSP-C in G if and only if there is an α-approximation algorithm for MCkCDSP in G'.*

Proof Clearly, the cost of a feasible solution D of an MCkCDSP-C instance $\langle G, \mathcal{C}, k\rangle$ is equivalent to the cost of D for an MCkCDSP instance $\langle G', k\rangle$ since in both problems, the cost of D is its size, i.e. the cardinality of D. By Theorem 4, an optimal solution O of MCkCDSP-C in G is a feasible solution of MCkCDSP in G'. Now, suppose O is not an optimal solution of MCkCDSP in G', and there exists another optimal solution O'. Then, by Theorem 4, O' is also a feasible solution of MCkCDSP-C in G. Since this contradicts to our initial assumption that O is an optimal solution of MCkCDSP-C in G, such a O' cannot exist. Therefore, an optimal solution O of MCkCDSP-C in G is an optimal solution of MCkCDSP in G'. Now, suppose we have an α-approximation algorithm $\mathcal{A}$ for MCkCDSP-C in G. Then, an output o of $\mathcal{A}$ satisfies $|o| \le \alpha|OPT| = \alpha|OPT_1|$, where OPT is an optimal solution of MCkCDSP-C in G and OPT_1 is an optimal solution of MCkCDSP in G'. As a result, $\mathcal{A}$ is also an α-approximation algorithm for MCkCDSP in G'. Using similar argument, we can prove an output o of an α-approximation algorithm for MCkCDSP in G' is also an α-approximation algorithm for MCkCDSP-C in G. As a result, this theorem is true.

5 Concluding Remarks and Future Work

In this paper, we study MCkCDSP, a new interesting optimization problem in social networks, and its variation MCkCDSP-C. After we show MCkCDSP is NP-hard, we introduced an approximation algorithm of MCkCDSP with $k = 2$. Furthermore, we also prove MCkCDSP-C is NP-hard and show an α-approximation algorithm of MCkCDSP can be used to have an α-approximation algorithm of MCkCDSP-C.

A l-connected graph G_l such that $l \geq k$ has a feasible solution of MkCDSP. However, with $l < k$, it is unclear if G_l still includes a subgraph which is a feasible solution of MkCDSP. In practice, a real social network graph consists of some very high degree nodes along with a large number of low degree nodes, each of which is connected to one or more high degree nodes. Therefore, it is very likely a feasible solution of MkCDSP in a social network graph exists in practice. Still, it is possible that a feasible solution may not exist in the graph. In such a case, we are forced to employ alternative approaches which are socially acceptable. For instance, we can start from $k = 1$, and compute a feasible solution of MkCDSP until we cannot find a feasible solution while increasing k by 1 each time we find a feasible solution. Alternatively, starting from a node u with low degree (and u, the residual graph is still connected), which usually represents a less important individual in our society, we try to find a feasible solution of MkCDSP in $G \setminus \{u\}$. If we cannot find a feasible solution, we remove the next less important node. This is repeated until we can find a feasible solution.

We admit that the approximation ratio of Algorithm 1 that we proved is not tight. For instance, in Fig. 3(a), the ratio of the size of an output of Algorithm 1 and the size of an optimal solution is much less than $3 \cdot (1 + 2\Delta) \cdot (3 + \ln \Delta)$. Therefore, it is possible that there is a significant gap between the lower bound of the performance ratio and what we achieved, and we plan to further investigate an approximation algorithm with tighter bound. We also plan to further study to reduce this gap and will investigate approximation algorithm for general $k > 2$.

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