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A PTAS FOR MINIMUM d-HOP UNDERWATER SINK PLACEMENT PROBLEM IN 2-D UNDERWATER SENSOR NETWORKS

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The multiple UnderWater Sink (UW-Sink) architecture is introduced to resolve the scalability problem in USNs. To maximize the benefit of this architecture, the UW-Sinks should be located carefully. Previously, we proposed Minimum d-Hop UW-Sink Placement Problem (MdHUWSPP) for time-sensitive applications, whose objective is minimizing the number of UW-Sinks which d-hop dominate a given USN. In this paper, we present a PTAS for MdHUWSPP. We introduce an algorithm to partition an input graph representing the USN, find local optimum solutions, and return the union of the

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local solutions. Later, we use this algorithm with the shifting technique to have a PTAS for MdHUWSPP.

Keywords: Polynomial time approximation scheme; underwater sensor networks; network center placement problem.

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1. Introduction

Nowadays, study about Underwater Sensor Networks (USNs) is popular due to their wide range of applications such as environmental monitoring, ocean sampling networks, etc [1]. USNs have several aspects which distinguish them from conventional Terrestrial Sensor Networks (TSNs). First, the cost of a durable underwater sensor node exceeds one thousand dollars. Second, USNs use acoustic channels instead of Radio Frequency (RF) signals, which cannot travel far in water. Meanwhile, acoustic communication links of USNs are volatile, have a longer propagation delay, and offer a very small bandwidth. Therefore, if a message is forwarded through more intermediate nodes, it may experience significantly longer data latency. As a result, a USN with a single UW-Sink becomes less efficient as the size of the network grows and is not scalable. Recently, multiple UnderWater-Sink (UW-Sink) architecture is introduced to resolve this problem in USNs [1, 2]. In this structure, each sensor node uses multihop horizontal links to send messages toward a UW-Sink. Each UW-Sink, a gateway to out-of-water network, has vertical links to forward messages to out-of-water sink which are usually ships or buoys. At last, those sinks outside water use RF signal or satellite links to communicate with each other or onshore stations.

Due to the special missions, UW-Sinks are expected to be very expensive. On the other hand, in USNs for time-sensitive applications, it is crucial to bound the data latency from sensors to UW-Sinks based on the requirements of applications. Therefore, once the minimum number of underwater sensor nodes are deployed using some existing strategy such as [3], it is very important to place UW-Sinks carefully. In [4], we studied the problem of designing cost-effective USNs for time-sensitive applications and proposed Minimum d-Hop UW-Sink Placement Problem (MdHUWSPP), whose objective is to find the minimum number of extra UW-Sinks for a given Unit Disk Graph (UDG) such that each node is at most d-hop away from its nearest UW-Sink. We also showed the NP-completeness of MdHUWSPP and proposed two constant factor approximation algorithms.

In this paper, we introduce the first Polynomial Time Approximation Scheme (PTAS) for MdHUWSPP. We introduce a primitive function MdHUWSPP-SUB and show how to obtain a PTAS for MdHUWSPP using MdHUWSPP-SUB. Given a reasonably small constant ϵ, we show that the Performance Ratio (PR) of our algorithm is $(1 + \epsilon)$ while its running time is polynomial. The rest of this paper is organized as follows. Section 2 introduce some related work. In Sec. 3, we introduce our PTAS for MdHUWSPP. At last, we make the conclusion in Sec. 4.

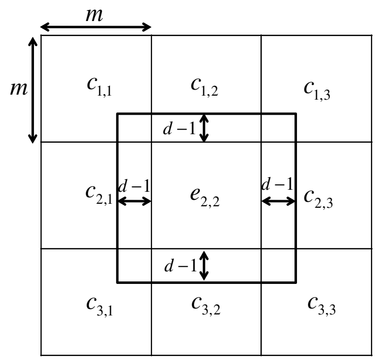

Fig. 1. $c_{2,2}$ is enlarged to $e_{2,2}$ by moving its each edge to outward by $d-1$ Euclidean distance.

2. Related Work

In [4], we proved that MdHUWSPP is NP-complete and presented two constant factor approximation algorithms. When $d = 1$, MdHUWSPP is equal to the Unit Disk Covering Problem (UDCP) in [5], which presented a PTAS for UDCP. Therefore, MdHUWSPP can be considered as the generalized version of UDCP. MdHUWSPP can be confused with Relay Node Placement Problem (RNPP) in [6] since their objectives are minimizing the number of extra nodes. However, in MdHUWSPP, new nodes are used to d-hop dominates existing nodes while in RNPP, new nodes are to connect existing nodes.

3. A PTAS for Minimum *d*-Hop UW-Sink Placement Problem

In this paper, $Eucdist(u, v)$ represents the Euclidean distance between two nodes v and u. $Hopdist(u, v)$ means the hop distance between two nodes v and u over the shortest path connecting them. A node u is d-hop dominated by a node v if $Hopdist(u, v) \leq d$. Finally, $N_d(v) = \{u|u \in V(G) \text{ and } Hopdist(u, v) \leq d\}$. We use an Unit Disk Graph (UDG) to model a 2-D USN. We assume that 1) the radius of each unit disk is one, 2) an input graph is connected, and 3) the global coordination of each node in the graph is known. Now, we introduce the formal definition of MdHUWSPP in [4].

Definition 3.1 (M*d*HUWSPP). Given a set of points $\{p_1, p_2, \ldots, p_n\}$ in the Euclidean plane and a fixed positive integer d, find a set of points H of minimum size in the Euclidean space such that for each p_i, there exists at least one $v \in H$ with $Hopdist(p_i, v) \leq d$.

Now we present a primitive function MdHUWSPP-SUB, which computes a local optimal solution for MdHUWSPP, and analyze its correctness and time complexity. Later, we show how to use MdHUWSPP-SUB to obtain a PTAS for MdHUWSPP and analyze the time complexity of the PTAS. Suppose t is the smallest integer

Algorithm 1 MdHUWSPP-SUB($G(V,E), RS, d, \epsilon, t, m$)

1: **for** $i = 1$ to t **do**
2: **for** $j = 1$ to t **do**
3: For $c_{i,j}$, build an expanded cell $e_{i,j}$ by moving each edge of $c_{i,j}$ to outward by $d-1$ Euclidean distance as in Fig. 1.
4: Prepare an empty set $H_{i,j}$. For each node pair $u, v \in e_{i,j}$ such that $Eucdist(u,v) \leq 2$, we can determine two unit disks C_1 and C_2. Put the centers of C_1 and C_2 into $H_{i,j}$. If there is a node v which does not have any node within Euclidean distance 2 from it, then put this node into $H_{i,j}$.
5: Compute the minimum number of nodes in $H_{i,j}$, denoted by $A_{i,j}$, which d-hop-dominates all nodes in $c_{i,j}$.
6: **end for**
7: **end for**
8: Return $A = \bigcup_{\forall i,j} A_{i,j}$

such that a square of size $tm \times tm$ can contain a given graph G for $m = 2\lceil \frac{3}{\epsilon} \rceil d$. Let us call this square RS. Make a grid of small squares on RS such that all cells in the grid are of equal size $m \times m$. The cell in the i-th row and the j-th column is denoted by $c_{i,j}$. Now, we introduce the MdHUWSPP-SUB algorithm for an input tuple (G, RS, d, ϵ).

Lemma 3.2. *For each cell c_{ij}, there exists an optimal solution for MdHUWSPP which is contained in H_{ij} constructed as in Step 4 of Algorithm 1.*

Proof. Suppose $Q = \{q_1, q_2, \ldots, q_l, \ldots, q_{|Q|}\}$ is an optimal solution to MdHUWSPP for $c_{i,j}$. Let S_l be a set of nodes in $c_{i,j}$ which are d-hop dominated by q_l. Let A_l be the set of nodes adjacent to q_l such that A_l can $(d-1)$-hop dominate S_l. By the construction of $e_{i,j}$, every node in A_l must lie in cell e_{ij}. First suppose that $|A_l| \geq 2$. Note if u, v are two points in A_l with $Eucdist(u,v) \leq 2$, then there exist at most two unit disk with its boundary passing through u and v. It follows that we can move q_l to a canonical position q'_l such that at least two nodes in A_l are on the boundary of a unit disk centered at q'_l and A_l is still covered by q'_l. If $|A_l| = 1$, the location of the only node in A_l is q'_l. Then, q'_l can still d-hop dominate all nodes in S_l. Therefore $Q' = \{q'_1, q'_2, \ldots, q'_{|Q|}\}$ is also an optimal solution to MCLP for the cell c_{ij} with $q'_l \in H_{ij}$. □

Lemma 3.3. *Step 3 ∼ 5 in Algorithm 1 computes an optimal solution of MdHUWSPP for each $c_{i,j}$ correctly and its time complexity is $(n^e_{i,j})^{O(m^2)}$, where $n^e_{i,j}$ is the number of nodes in $e_{i,j}$.*

Proof. First, we claim that we need at most $\lceil \sqrt{2}m \rceil^2$ UW-Sinks to d-hop-dominate all nodes in $c_{i,j}$. Suppose $c_{i,j}$ is partitioned into smaller regular squares with edge

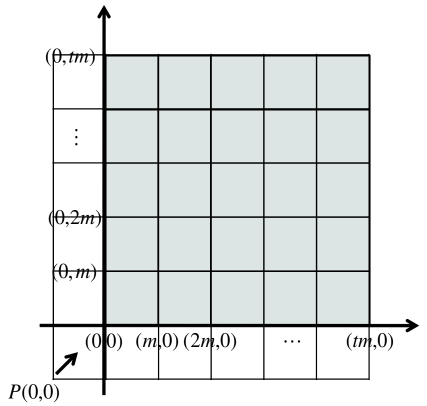

Fig. 2. Partition and shifting techniques.

length $\mu = \frac{\sqrt{2}}{2}$. If we put one UW-Sink in each small cell, then all nodes $e_{i,j}$ will be 1-hop (and hence d-hop) dominated by some UW-Sink. As a result, at most $\lceil \sqrt{2}m \rceil^2$ UW-Sinks are required.

By Lemma 3.2, we can move nodes in Q to some canonical positions whose number is at most $2\binom{n_{i,j}^e}{2}$. Therefore, an optimal solution of MdHUWSPP for $c_{i,j}$ is a subset of $H_{i,j}$ in Step 5 of Algorithm 1 and we have at most

$$\sum_{k=0}^{\lceil \sqrt{2}m \rceil^2} \binom{n_{i,j}^e(n_{i,j}^e - 1)}{k} = O((n_{i,j}^e)^{4m^2})$$

possibilities. For each case, we need to verify its correctness within $O(m^2 n_{i,j}^e)$ time, and thus the total time complexity is $O(m^2 n_{i,j}^e) O((n_{i,j}^e)^{4m^2}) = (n_{i,j}^e)^{O(m^2)}$. □

Now, we are going to use the shifting technique similar to the one in [5] to show the performance ratio of MdHUWSPP-SUB.

Theorem 3.4. *There exists a PTAS for MdHUWSPP with time complexity* $n^{O(1/\epsilon^2)}$.

Proof. In Algorithm 1, let $m' = \lceil \frac{3}{\epsilon} \rceil$. Then, $m = 2m'd$. Assume that $RS = \{(x,y) | 0 \leq x \leq tm, 0 \leq y \leq tm\}$. Define $\overline{RS} = \{(x,y) | -m \leq x \leq tm, -m \leq y \leq tm\}$ and partition $\overline{RS}$ into $(t+1) \times (t+1)$ grids such that each cell in $\overline{RS}$ is an $m \times m$ regular square. Here, we assume the top and right boundaries do not belong to the cell, so that all cells do not overlap and no d-hop neighborhood $N^d(v)$ can intersect with three parallel strips. Denote this $\overline{RS}$ by $P(0,0)$. Generally, denoted by $P(k,k)$ the partition of $\overline{RS}$, which is obtained from $P(0,0)$ by moving the bottom-left corner of $P(0,0)$ from $(-m,-m)$ to $(-m+2dk, -m+2dk)$, for

$k = 0, 1, \dots, m' - 1$ (see Fig. 2). At last, we compute

$$A^{(k)} = \text{M}d\text{HUWSPP-SUB}(G(V,E), P(k,k), d, \epsilon, t, m)$$

for all k and pick the best result among them.

Now, we show that $|A^{(k)}| \le (1+\epsilon)|OPT^*|$ for arbitrary small $\epsilon > 0$ and a proper k, where OPT^* is an optimal solution to MdHUWSPP in the whole input graph. Let $e_{i,j}$ be a cell in $P(k,k)$ and

$$OPT^*_{e_{i,j}} = \{v \in OPT^* \mid v \ d\text{-hop dominates some nodes in } e_{i,j}\}.$$

Clearly each node in $e_{i,j}$ is d-hop dominated by $OPT^*_{e_{i,j}}$. It follows that $|A^{(k)}_{e_{i,j}}| \le |OPT^*_{e_{i,j}}|$. Thus we have

$$|A^{(k)}| = \left| \bigcup_{\forall i,j} A^{(k)}_{e_{i,j}} \right| \le \sum_{\forall i,j} |A^{(k)}_{e_{i,j}}| \le \sum_{\forall i,j} |OPT^*_{e_{i,j}}|.$$

Let H_k (resp. V_k) be the subset of nodes in OPT^* such that $v \in H_k$ if and only if all nodes in $N^d(v)$ lie exactly in two successive horizontal (resp. vertical) strips of $P(k,k)$. Note that no d-hop neighborhood of a node lies in three successive horizontal or vertical strips. Let's count the number of elements in OPT^* through counting that of $OPT^*_{e_{i,j}}$ for all i and j. Note that nodes in $H_k \setminus V_k$ and $V_k \setminus H_k$ will be counted at most twice and nodes in $H_k \bigcap V_k$ will be counted at most four times. Thus we have

$$\sum_{\forall i,j} |OPT^*_{e_{i,j}}| \le |OPT^*| + |H_k| + 2|V_k|.$$

Further note that all H_k are disjoint for $k = 0, 1, \dots, m' - 1$. It follows that $\sum_{k=0}^{m'-1} |H_k| \le |OPT^*|$. Similarly, we have $\sum_{k=0}^{m'-1} |V_k| \le |OPT^*|$. Thus,

$$\sum_{k=0}^{m'-1} |A^{(k)}| \le \sum_{k=0}^{m'-1} (|OPT^*| + |H_k| + 2|V_k|) \le (m' + 3)|OPT^*|,$$

from which we obtain that $\frac{1}{m'} \sum_{k=0}^{m'-1} |A^{(k)}| \le (1 + \frac{3}{m'})|OPT^*|$. Since $m' = \lceil \frac{3}{\epsilon} \rceil$, the inequality above implies that there exists at least one k such that $|A^{(k)}| \le (1+\epsilon)|OPT^*|$. Let's consider the time complexity of our scheme. Since each $c_{i,j}$ is surrounded by at most 8 other cells, and cell $e_{i,j}$ is contained in the union of $c_{i,j}$ and the other cells around it. From Lemma 3.3, the time complexity of MdHUWSPP-SUB is

$$\sum_{\forall i,j} (n^e_{i,j})^{O(m^2)} \le \left(\sum_{\forall i,j} n^e_{i,j} \right)^{O(m^2)} \le (9n)^{O(m^2)} = n^{O(m^2)}.$$

Since we execute MdHUWSPP-SUB for m' times, the complexity of our scheme is $m' n^{O(m^2)} = n^{O(\frac{1}{\epsilon^2})}$ and our scheme is a PTAS for MdHUWSPP. □

4. Conclusion

In this paper, we proposed the first PTAS to solve MdHUWSPP. In our algorithm, we divide whole graph into the smaller pieces and find an optimal solution of MdHUWSPP in each of them. Using shifting technique, we find an approximated solution and bound the error between the union of local optimum solutions and a global optimum solution. At last, we show that this algorithm is in fact a PTAS for MdHUWSPP. As a future work, we are interested in variations of MdHUWSPP by considering energy model, etc.

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